Experimentální Analýza Napětí 2011
Experimental Stress Analysis

Applying High Order Statistics Analysis at Non Destructive Evaluation of Concrete Tiles

Lubos Pazdera[1], Jaroslav Smutny[2] & Libor Topolar[3]

Abstract: The paper submitted deals with the investigation of failures of engineering structures by means of the Method of higher orders spectra. The reason why Method of higher orders spectra is prospective in future is that unlike other non-destructive methods this one offers a high-quality information on the existence of non-linear processes in material and geometrically complex investigated samples (structures) and thus also the information on the existence and degree of development of particular faults. A timely detection of the level or location of a failure of a structural member or of the entire structure is gaining its importance at present due to the increasing maintenance costing. The application of this method will also offer the source materials about real material and geometrical characteristics of engineering structures in existing numerical models.

Keywords: Experiment, High Order Spectra, Tile, Non Destructive Testing, Lifetime, Structure Defect, Civil Engineering, Analysis, Non Linear Analysis

1. Introduction

It has appeared recently that there is a lack of simple non-destructive methods applicable to a fast analysis of different structure elements or whole structures, which is typical particularly of the field of civil engineering. This has been demonstrated by tile structure breakdowns as well as many collapses of house and hall roof structures in 2006 in consequence of excess weight overload due to a large quantity of snow lying on them. [1]

Another expected problem area comprises bridge structures and for example also prefabricated panel houses. Among marked factors affecting the condition of building structures, there are the following: poor maintenance, inappropriate service load, unfavourable geologic conditions, climatic effects, etc. Another cause may consist in the design, production process, construction procedures or operating conditions. Depending on the building type, the critical condition of structures (and the resulting need for rehabilitation or reconstruction) comes after some 30 years of service. At present, there is a range of different methods designed to non-destructive testing of materials, building elements and entire structures. Methods based on the elastic wave propagation, reflection and interference appear to be currently in the technical public's limelight. Among them there are: acoustic emission, Impact-echo, or some non-linear ultrasonic spectroscopy methods, but also vibration analysis or modal analysis methods, to mention them in brief. All of these methods require sophisticated instrumentation and highly skilled operating personal. Some of them fail to

[1] *prof. Ing. Lubos Pazdera, CSc, Brno University of Technology, Faculty of Civil Engineering, Department of Physics, Veveri 331/95, Brno, Czech Republic, Pazdera.L@fce.vutbr.cz*
[2] *prof. Ing. Jaroslav Smutny, PhD, Brno University of Technology, Faculty of Civil Engineering, Department of Raylway Structures and Constructions, Veveri 331/95, Brno, Czech Republic, Pazdera.L@fce.vutbr.cz*
[3] *Mgr. Libor Topolar, PhD, Brno University of Technology, Faculty of Civil Engineering, Department of Physics, Veveri 331/95, Brno, Czech Republic, Topolar.L@fce.vutbr.cz.*

provide plausible results for complicated-shape building elements or inhomogeneous-material-made building elements. [1]

This is why it is imperative to look permanently for simple, reliable and fast-operating diagnostic methods being based on new achievements in the signal analysis and material engineering fields. In what follows, we are going to describe an application of the Higher-order-spectrum method to the roofing tile analysis. [2]

2. Theoretical introduction

Higher Order Statistics is extension of second-order characteristics such as the auto-correlation function and power spectrum to higher orders. Higher Order Statistics Analysis is emerging as a new powerful technique in signal analysis, offering insight into non-linear coupling between frequencies and potential applications in many areas where traditional linear analysis provides insufficient information.

The second-order analyses work fine if the signal has a Gaussian (Normal) probability density function, but many real-life signals are non-Gaussian. The easiest way to introduce the Higher Order Statistics measures is just to show some definitions so that the reader can see how they are related to the familiar second-order measures. In the text to follow, are definitions for the time-domain and frequency-domain third-order Higher Order Statistics measures, moments, cumulants, high-order spectra or polyspectra, etc.

2.1. Moments

Statistical moments are a set of quantities, describing in a simplified manner the shape of the probability function. The first-order moment, m_1, is the mean value of the process $x(t)$.

$$m_1 = E\{x(t)\} \tag{1}$$

The second-order moment, m_2, is its dispersion.

$$m_2(\tau) = E\{x(t) \cdot x(t+\tau)\} \tag{2}$$

Higher-order moments are the acuteness, m_3, skewness,

$$m_3(\tau_1, \tau_2) = E\{x(t) \cdot x(t+\tau_1) \cdot x(t+\tau_2)\} \tag{3}$$

m_4, etc.

$$m_4(\tau_1, \tau_2, \tau_3) = E\{x(n) \cdot x(n+\tau_1) \cdot x(n+\tau_2) \cdot x(n+\tau_3)\} \tag{4}$$

Special non-linear combinations of these moments are so-called cumulants. The process first-order cumulant, c_1, is the process mean value. The second order, c_2, third-order c_3 and fourth-order c_4 process cumulants are defined by following equations [3]:

$$c_1 = m_1 \tag{5}$$

$$c_2 = m_2 - m_1^2 \tag{6}$$

$$c_3 = m_3 - 3 \cdot m_2 \cdot m_1 + 2 \cdot m_1^3 \tag{7}$$

$$c_4 = m_4 - 4 \cdot m_3 \cdot m_1 + 3 \cdot m_2^2 + 12 \cdot m_2 \cdot m_1^2 - 6 \cdot m_1^4 \tag{8}$$

2.2. Polyspectra

Polyspectra is used to describe the family of all frequency-domain spectra, including those of the second-order. Most high order statistics work based on polyspectra and focus their attention on the bispectrum (third-order polyspectrum) and the trispectrum (fourth-order polyspectrum). Polyspectra consist of higher order moment spectra and cumulant spectra and can be defined for both the deterministic signal and random processes. Moment spectra can be very useful in the analysis of deterministic signals (transient and periodic), whereas cumulant spectra can play a very important role in the analysis of stochastic signals. The k^{th}-order polyspectrum is defined as the Fourier transform of the corresponding cumulant sequence [3]:

$$C_2(f) = \sum_{k=-\infty}^{\infty} c_2(k) \cdot \exp(-i2\pi f k) \tag{9}$$

$$C_3(f_{1,} f_2) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} c_3(k,l) \cdot \exp(-i2\pi f_1 k) \cdot \exp(-i2\pi f_2 l) \tag{10}$$

$$C_4(f_1, f_2, f_3) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_4(k,l,m) \cdot \exp[-i2\pi(f_1 k + f_2 l + f_3 m)] \tag{11}$$

which are the power spectrum, respectively the bispectrum and trispectrum. These can be estimated in a way similar to the power spectrum, but more data is usually needed to get reliable estimates. Note that the bispectrum is a function of two frequencies, whereas the trispectrum is a function of tree frequencies. In contrast to the power spectrum which is real-valued and non-negative, bispectrum and trispectrum are complex valued and contain a phase information.

For a real-valued process, symmetry properties of cumulants carry over to symmetry properties of polyspectra. The power spectrum is symmetric [6]

$$C_2(f) = C_2(-f) \tag{12}$$

The symmetry properties of the bispectrum are given :

$$\begin{aligned} C_3(f_1, f_2) &= C_3(f_2, f_1) = C_3(f_1, -f_1 - f_2) = C_3(-f_1 - f_2, f_2) = \\ &= C_3^*(-f_1, -f_2) \end{aligned} \tag{13}$$

Symmetry properties of the trispectrum include:

$$\begin{aligned} C_4(f_1, f_2, f_3) &= C_4(f_1, f_3, f_2) = C_4(f_2, f_1, f_3) = \\ &= C_4(-f_1, f_2 - f_1, f_3 - f_1) = C_4^*(-f_1, -f_2, -f_3) \end{aligned} \tag{14}$$

2.3. Bispectrum

The bispectrum can be calculated in a similar way as the Welch's periodogram for the power spectrum estimate. It is advisable for the data sequence, required for obtaining a consistent estimate, to be Langer that that for the power spectrum estimate calculation. The estimate dispersion can be reduced in amount by averaging over the date segment multiples. [4]

$$B(f_1, f_2) = \frac{\left|C_3(f_1, f_2)\right|^2}{\sqrt{P(f_1) \cdot P(f_2) \cdot P(f_1 + f_2)}} \tag{15}$$

where $C_3(f_1,f_2)$ is the bispectrum estimate and $P(f)$ is the power spectrum estimate.

The power spectrum is the primary tool of signal processing, and algorithms for estimating the power spectrum have found applications in areas such as radar, sonar, seismic, biomedical, communications, and speech signal processing. [6]

3. Experimental result

The mathematical formalism based on the Higher-order spectrum method, being elaborated and optimized in our research, was step-by-step verified in laboratory environment, by testing selected structural element assemblies. To verify the above mentioned method, simple-geometry and simple-material-made structural elements, both flawless and containing intentionally simulated geometry and structure related defects, have been tested. In this paper, there are presented test results for a flawless concrete roof tile and for a roof tile containing simulated defect, intentionally created crack.

An impulse exciting signal response method was used to the specimen testing. An impact hammer was used to excite the specimen. The monitoring frequency interval was specified to range up to 10 kHz. A PULSE 3560C system by the firm Brüel& Kjaer was used to control the experiments. The response signals were measured by accelerometer sensors, which were fixed to the specimen at preselected points.

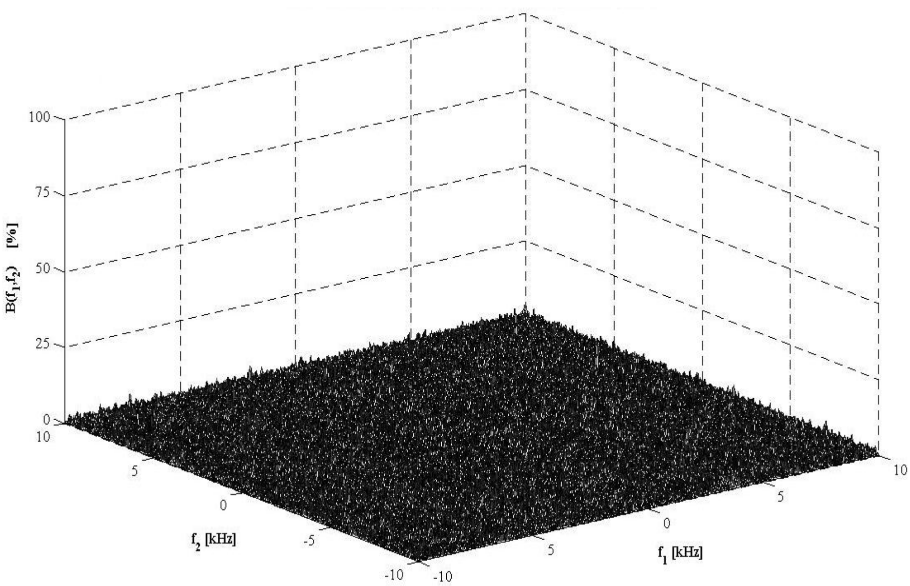

Fig. 1. Bispectrum of concrete tile without defect

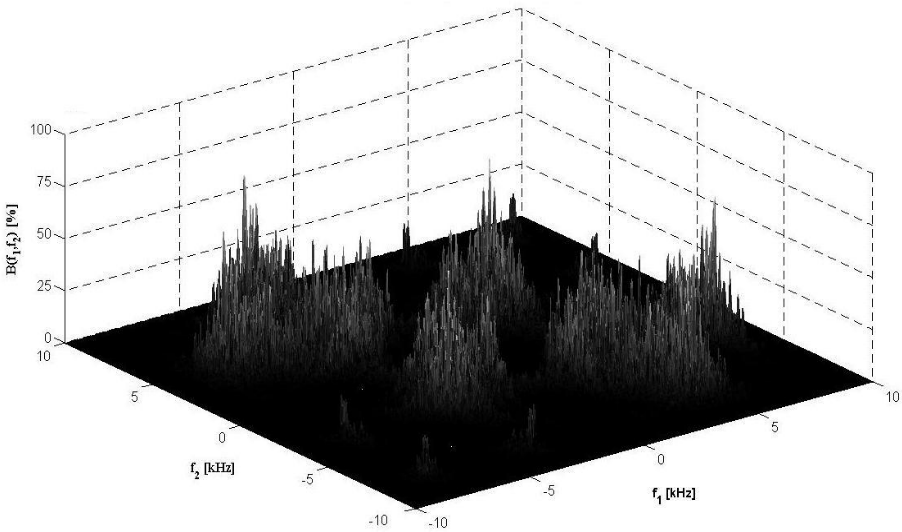

Fig. 2. Bispectrum of concrete tile with defect

The graphs in Figs. 1 and 2 represent impulse responses of the tiles without (Fig. 1) and with (Fig. 2) defect. It is evident from this graphs that particular frequency peaks contained mainly lower values for tile without defect. Consequently great differences between “good” and “bad” tile are evident if the graphs of bicoherence.

4. Conclusion

The field of higher-order statistics (spectra) and its applications to various signal processing problems are relatively new. [5]

In conclusion, it may be stated, based upon the measurements and analyses carried out that the experiment carried out successfully verified the possibilities of utilising the given method for assessing the quality of particular components of the building structure. It follows from the text mentioned above that the method of higher order spectra may play an important role in future when analysing quality of structures.

Ascertained fact is that this method can be used to detect structural defects and to assess the quality of the structure, which is not essentially made possible when both the calculation models and the typical methods used until now are applied. Laboratory tests should also be verified by measurements in the field.

Acknowledgement

This research has been supported by project of GACR No. P104/10/0535.

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