Experimentální Analýza Napětí 2011
Experimental Stress Analysis

Determination of Poisson's Ratio of Gypsum Materials

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Abstract: Poisson's ratio is one of the basic material characteristics. For some methods, e. g. nanoindentaion, the Poisson's ratio is one of the parameters, which are needed for calculation of Young's modulus for tested indented material. One possibility how to determine Poisson's ratio of the gypsum specimen is the use of nondestructive method based on resonant frequency measurement. The Poisson's ratio was determined for dental gypsum specimens made with six different water/gypsum ratios.

Keywords: Poisson's ratio, Gypsum, Dynamic Young's modulus, Water / gypsum ratio

1. Introduction

At present time, gypsum is usually used for interior applications as blocks for bathroom walls, plasterboards or as fire safeguards [1] but there are also some applications where special types of modified gypsum are needed. The next well known application of the gypsum is in medicine as fixation of broken arms and legs or in stomatology.

The dental gypsum is a little bit different from classic gypsum used in building industry. The strength of the dental gypsum is much higher than strength of classic gypsum. Table 1 shows types of dental gypsum. Every type of gypsum is described from the point of view of compressive strength of hardened gypsum and expression during/after hardening. The table concludes expansion rates during hardening and usual value of water/gypsum ratio.

Table 1. Types of dental gypsum

Type of gypsum	Description	Expansion rate [%]	Water/gypsum ratio
Type I	Expression plaster	0.15	0.50
Type II	Model plaster	0.30	0.50
Type III	Hard stone	0.20	0.30
Type IV	Super hard stone (low expansion)	0.10	0.22
Type V	Super hard stone (high expansion)	0.30	0.22

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The main aim of this work was to determine the dependency of the dynamic Young's modulus, the dynamic shear modulus and the Poisson's ratio on the water/gypsum ratio of the high strength gypsum.

2. Material and specimens

The commercially available gypsum binder Interdent® on a dental application was used for testing the dependence between water/gypsum ratio and mechanical properties of gypsum samples. This gypsum binder is an extra hard material for precision techniques during the construction of porcelain crown sand and bridges. The dental gypsum binder is a material with high resistance to compression (resistance after 24 hours is 250 MPa for a water/gypsum ratio 0.2) with high surface density, resistance to abrasion and low setting expansion (0.1 %).

It was assumed that water to gypsum (w/g) ratio would have influence on its mechanical properties. Therefore, six different types of specimens with w/g = 0.14, 0.18, 0.19, 0.20, 021 and 0.22, were prepared. During preparing gypsum specimens, amount of the gypsum binder and amount of water which corresponded with used water/gypsum ratio (in our case five different values) were weighted at first. Than the gypsum was slowly added into the water and vigorously hand mixed with a spade approximately 1 minute until the mass was smooth. The mass was cast into a shape and 30 second vibrated on the vibration table. The specimens were removed from the mould twenty four hours after mixing. The fourteen specimens of dimensions 40 × 40 × 160 mm with different water to gypsum ratios were prepared and tested.

3. Impulse excitation method

The impulse excitation method was used for determination of the dynamic Young's modulus, the dynamic shear modulus and the Poisson's ratio. The method is based on measuring fundamental resonant frequencies of longitudinal, transversal and torsional vibration of the specimens.

3.1. Longitudinal vibration

The specimen was supported in the middle of its span (Fig. 1), the fundamental longitudinal nodal position. The acceleration transducer Bruel&Kjaer of Type 4519-003 was placed at the centre of one of the end faces of the gypsum specimen (Fig. 1- the right end face). The end face of the gypsum specimen opposite to the face, where the transducer was located, was struck by the impact hammer Bruel&Kjaer of Type 8206 (Fig. 1- on the left). Both signals, the excitation force and the acceleration, were recorded and transformed using Fast Fourier Transform (FFT) to the frequency domain, and the Frequency Response Function (FRF) was evaluated from these signals using the vibration control station Bruel&Kjaer Front-end 3560-B-120 and program PULSE 10.5. The test was repeated five times for each gypsum specimen and resultant readings were averaged. From an averaged FRF, the fundamental longitudinal resonant frequency was determined for each specimen.

Fig. 1. The impulse excitation method – test arrangement for measurement of fundamental longitudinal resonant frequency

3.2. Transversal vibration

The specimen was simply supported in the distance $0.224 \times l$ of the span on both ends (Fig. 2), the transversal nodal positions of the first mode of transversal vibration. The acceleration transducer Bruel&Kjaer of Type 4519-003 was placed at the end of the specimen on the upper face (Fig. 2- the right end). The upper surface of the opposite end of the specimen was struck by the impact hammer Bruel&Kjaer of Type 8206. The first transversal resonant frequency was evaluated using the same procedure as the above described longitudinal one.

Fig. 2. The impulse excitation method – test arrangement for measurement of fundamental transversal resonant frequency

3.3. Torsional vibration

The specimen was supported in the middle of its span (Fig. 3), the torsional nodal position. The acceleration transducer Bruel&Kjaer of Type 4519-003 was placed at the end of the specimen in the right upper corner of the side face (Fig. 3- the right end). The left lower corner of the same side face of the specimen was struck by the impact hammer Bruel&Kjaer of Type 8206. The first torsional resonant frequency was evaluated using the same procedure as the above described longitudinal one.

Fig. 3. The impulse excitation method – test arrangement for measurement of fundamental torsional resonant frequency

4. Evaluation of dynamic material properties

The dynamic Young's modulus was determined twice, firstly based on the longitudinal resonant frequency and secondly based on the transversal resonant frequency [2], [3]. The dynamic shear modulus was determined based on the torsional resonant frequency and Poisson's ratio was calculated from dynamic Young's modulus and dynamic shear modulus.

Based on the equation for longitudinal vibration of the beam with continuously distributed mass with free-free boundary condition, the dynamic Young's modulus E_{dl} can be determined using the relation

$$E_{dl} = \frac{4lmf_l^2}{bt} \tag{1}$$

where l is the length of the specimen [m], m is the mass of the specimen [kg], f_l is the fundamental longitudinal resonant frequency of the specimen [Hz], b is the width of the specimen [m] and t is the thickness of the specimen [m].

The dynamic shear modulus G_d can be determined based on the equation

$$G_d = \frac{4lmf_t^2}{bt}[B/(1+A)] \tag{2}$$

where f_t is the fundamental torsional resonant frequency of the specimen [Hz], A is an empirical correction factor dependent on the width-to-thickness ratio of the specimen defined in [2] and B is defined as

$$B = \left[\frac{b/t + t/b}{4(t/b) - 2.52(t/b)^2 + 0.21(t/b)^6}\right] \tag{3}$$

The Poisson's ratio μ_{dl} can be calculated from

$$\mu_{dl} = (E_{dl}/2G_d) - 1 \tag{4}$$

and then the dynamic Young's modulus E_{df} based on transversal resonant frequency can be determined using the relation

$$E_{df} = \frac{0.9465 l^3 m f_f^2 T_1}{bt^3} \tag{5}$$

where f_f is the fundamental transversal resonant frequency of the specimen [Hz] and T_1 is correction factor for fundamental flexural mode to account for finite thickness of bar, Poisson's ratio, and so forth. It is defined in [2].

Because the correction factor T_1 is dependent on μ_d, an iterative process has to be used for μ_{df} and E_{df} determination. As the first approximation of μ_{df} for calculation E_{df}, we used μ_{dl} as mentioned above and then the next μ_{df} can be calculated

$$\mu_{df} = (E_{df}/2G_d) - 1 \tag{6}$$

The iterative process of equations (5) and (6) stoped when the values of μ_{df} for the previous and the next steps did not differ more than 1%.

5. Results

At first, the specimens with water/gypsum ratios 0.18, 0.19, 0.20, 0.21 and 0.22 were made and tested. Then the three new sets of specimens with 0.14, 0.18 and 0.22 were made and tested, each set consisted of three specimens. The Fig. 4 shows the evaluated dynamic Young's modulus determined based on the longitudinal resonant frequency E_{dl} and based on the transversal resonant frequency E_{df}. The Fig. 5 shows the values of the evaluated dynamic shear modulus. The graph of the Poisson's ratio μ_d is visible in Fig. 6.

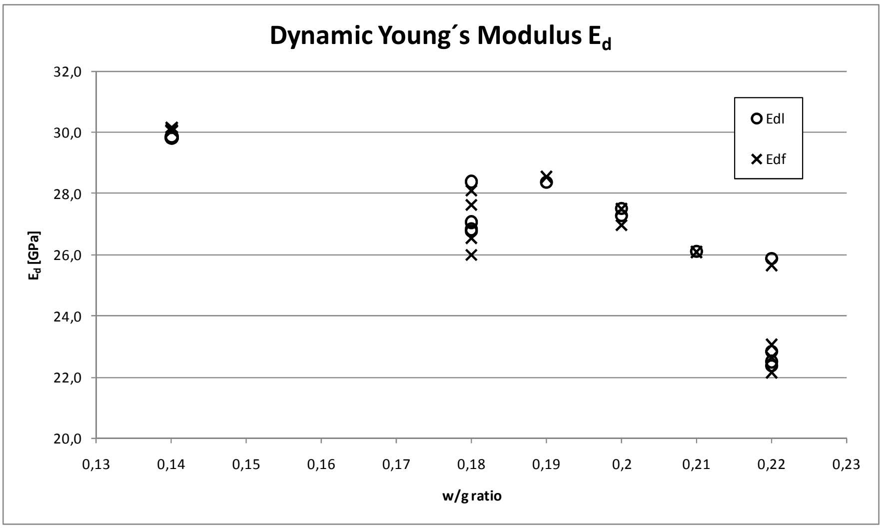

Fig. 4. The dynamic Young's modulus determined based on the fundamental longitudinal resonant frequency E_{dl} and based on the fundamental transversal resonant frequency E_{df}

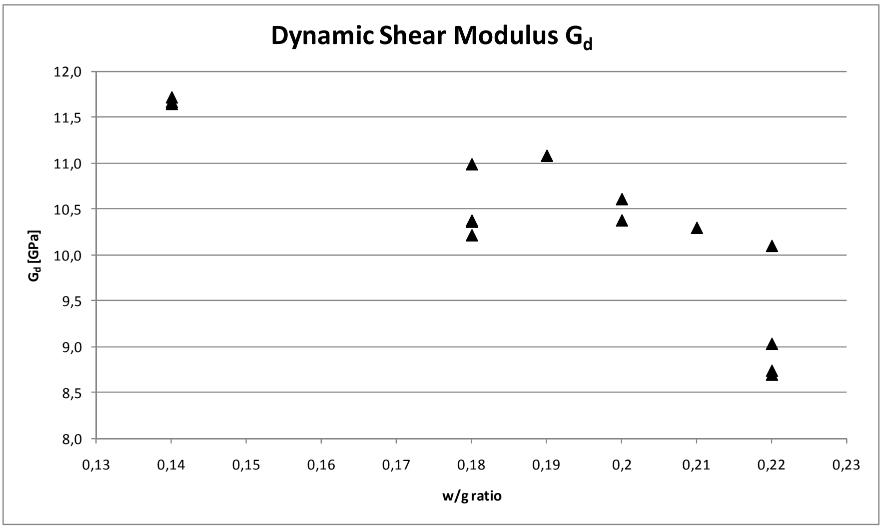

Fig. 5. The dynamic shear modulus G_d determined based on the fundamental torsional resonant frequency

6. Conclusion

The paper deals with measurement of dynamic material properties: dynamic Young's modulus, dynamic shear modulus and Poisson's ratio, and with the determination of the influence of water/gypsum ratio on these characteristics. It can be seen that the dynamic Young's modulus and dynamic shear modulus decreased in dependence on increase of the used water/gypsum ratio of the dental gypsum specimens (Fig. 4 and Fig. 5). This dependency is well known, also other characteristics (e.g. compressive strength, tensile strength in three point bending test) are changing in dependence on w/g ratio. In our further work, we will test more specimens to determine this dependence more accurately. The measured E_d values for one value of w/g ratio are quite different but the character of the dependence between E_d and w/g ratio is still visible (Fig. 4). The determined values of Poisson's ratio μ_d (Fig. 6) show that the Poisson's ratio μ_d is not dependent on water/gypsum ratio, what is visible from relation (6), because it is ratio of two moduli which are dependent on w/g ratio (Fig. 4 and Fig. 5).

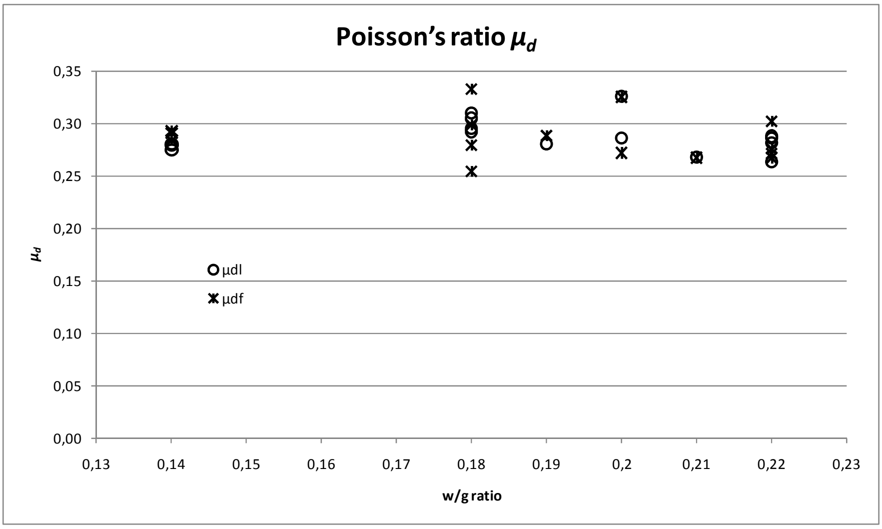

Fig. 6. The Poisson's ratio μ_d determined based on longitudinal and torsional vibration μ_{dl} and based on the transversal and torsional vibration μ_{df}

Acknowledgements

This paper has been supported by the Ministry of Education of the Czech Republic under the No.: MSM 6840770031

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