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Experimental Stress Analysis

Regression Model of the Hole Drilling Principle for the Stress State Identification

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Abstract: The idea of the presented numerical simulation technique corresponds to the E 837 standard concepts but is more universal. It transforms the strains, arising during the hole drilling experiment, in a way similar to that of the E 837 standard but, unlike the E 837 standard, it executes the transformation completely. This theory enlargement enables the drilling method to be applied for a wider spectrum of further measuring appliances. Moreover, the hole drilling process do not have to be extremely precise, which the whole procedure simplifies, since the new method principle includes an objective stress state identification, when evaluating drilling experiments, with respect to the drilled hole eccentricity. The identification method of the regression parameters of the hole drilling principle are derived in this paper.

Keywords: Stress state identification, Hole drilling method, Regression parameters

1. Introduction

The hole-drilling experimental method for stress state identification results in a small cylindrical hole drilled into an examined component surface. The hole drilling method for the stress state identification is based on the assumption that the free surface is one of the principal planes. The stress state in the surface layer thus can be only a uniaxial or a plane one. As such, it should be identifiable by measuring strains relieved on the free surface of the pre-strained structure during the drilling of a hole perpendicular to the surface. The semi-destructive hole drilling principle [4, 5] is based on impairing of the inner force equilibrium of a strained structure by drilling a relatively small circular hole perpendicularly to the surface. A drilled hole induces a change of the strain state in its close vicinity. These changes can be adjusted to define the strains arisen by drilling and thus used later for an identification of the original strain state after the strains relieved by the drilled hole are measured. If the hole of the radius R_0 has not been drilled yet, the thin plate depicted in Fig. 1, which is loaded uni-axially by principal stress σ_x, is loaded by stresses σ'_r, σ'_Θ, $\tau'_{r\Theta}$ in planes defined by the ratio radius $r = R/R_0$ and α polar coordinates and marked by indices of their normal lines r, Θ, which are determined in Eq. 1. The theory of the hole drilling principle is based on the analytical Kirsch's stress-state solution of a plate with a hole drilled through perpendicularly and loaded on its x-borders by principal stress σ_x [1]. The Kirsch's equations (Eq. 2) describe the state of plane stress in the vicinity of the through hole of radius R_0 (Fig. 1). In comparison with Eq. (1), Eq. (2) include terms dependent on the drilled through hole, which are left in Eq. (3) that are otherwise of a character similar to Eq. (1) and (2). If E stands for Young's modulus and ν for Poisson's ratio, the changes of plane stresses σ_r, σ_Θ, $\tau_{r\Theta}$ can be used for any isotropic material for a calculation of changes related to strains ε_r, ε_Θ, $\gamma_{r\Theta}$ and ε_z (see Fig. 1) in a point on the plate using Hooke's law.

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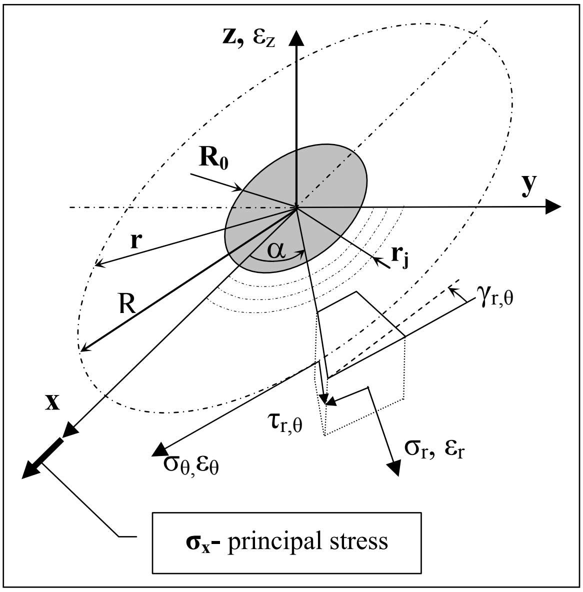

Fig. 1. State of stress and strains around the drill hole

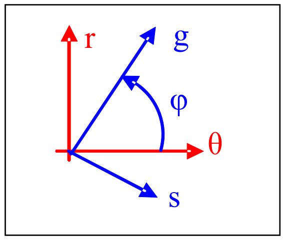

Fig. 2. Winding angle

$$\left\{\begin{aligned} \sigma'_r &= \frac{\sigma_x}{2}(1+\cos 2\alpha) \\ \sigma'_\theta &= \frac{\sigma_x}{2}(1-\cos 2\alpha) \\ \tau'_{r\theta} &= \frac{\sigma_x}{2}\sin 2\alpha \end{aligned}\right\} \quad (1)$$

$$\left\{\begin{aligned} \sigma''_r &= \frac{\sigma_x}{2}(1-\frac{1}{r^2}) + \frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sigma''_\theta &= \frac{\sigma_x}{2}(1+\frac{1}{r^2}) - \frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha \\ \tau''_{r\theta} &= \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{aligned}\right\} \quad (2)$$

$$\left\{\begin{aligned} \sigma''_r - \sigma'_r &= \frac{\sigma_x}{2}(-\frac{1}{r^2}) + \frac{\sigma_x}{2}(\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sigma''_\Theta - \sigma'_\Theta &= \frac{\sigma_x}{2}(\frac{1}{r^2}) - \frac{\sigma_x}{2}(\frac{3}{r^4})\cos 2\alpha \\ \tau''_{r\Theta} - \tau'_{r\Theta} &= \frac{\sigma_x}{2}(-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{aligned}\right\} \quad (3)$$

$$\left\{\begin{aligned} \sigma_r(c_1,c_2,c_3) &= \frac{\sigma_x}{2}(-\frac{1\cdot c_1(r,\alpha,z)}{r^2}) + \frac{\sigma_x}{2}(\frac{3\cdot c_2(r,\alpha,z)}{r^4} - \frac{4\cdot c_3(r,\alpha,z)}{r^2})\cos 2\alpha = \bar{\sigma}_r \\ \sigma_\Theta(c_4,c_5) &= \frac{\sigma_x}{2}(\frac{1\cdot c_4(r,\alpha,z)}{r^2}) - \frac{\sigma_x}{2}(\frac{3\cdot c_5(r,\alpha,z)}{r^4})\cos 2\alpha = \bar{\sigma}_\Theta \\ \tau_r(c_6,c_7) &= \frac{\sigma_x}{2}(-\frac{3\cdot c_6(r,\alpha,z)}{r^4} + \frac{2\cdot c_7(r,\alpha,z)}{r^2})\sin 2\alpha = \bar{\tau}_r \end{aligned}\right\} \quad (4)$$

The use of the hole drilling method for identification of residual stresses [2] is supported by E 837 standard [3]. The measured strain (radial ε_r or tangential ε_Θ) on the surface near the drilled hole is realized by strain gauges assembled to a drilling rosette. These strain components and analogously stress components are similar to the Kirsch's solution of the thin plate with a through hole described in Eq. (3). A simplification of goniometric functions Eq. (3) describing a response of an ideal strain gauge placed with a deviation of angle α from a direction related to the principal stress σ_x is used (see Eq. (5)). Standard constant variables $\overline{A} = -\overline{a}(1+\nu)/2E$ and $\overline{B} = -\overline{b}/2E$ related to the particular design of the drilling rosette are used within the superposition of principal stresses σ_x and σ_y looked for. The $\overline{a}$ constant is objectively independent of the material drilled, while constant $\overline{b}$ is here simplified because it is mildly dependent on Poisson's ratio ν of Hooke's material (see Eq. (6)). The two constants $\overline{a}, \overline{b}$ are tabulated in E 837 standard for particular types of drilling rosettes, the ratio of diameters $1/r = 2R_0/2R$ given and the relative depth of the drilled hole $z/2R$, where z is the depth of the hole.

$$\varepsilon_r = \sigma_x(\overline{A} + \overline{B}\cos 2\alpha) + \sigma_y(\overline{A} + \overline{B}\cos(2\alpha + \pi)) = \sigma_x(\overline{A} + \overline{B}\cos 2\alpha) + \sigma_y(\overline{A} - \overline{B}\cos 2\alpha) \qquad (5)$$

Strain gauge rosette sizes are comparable with those of the drilled hole diameters $2R_0$ or middle radii R, at which the strain gauges of the rosettes are placed. The measuring properties of the rosettes during the hole drilling according to E 837 standard are considerably dependent on the accuracy of compliance with standardized conditions of the experiment. If the hole is drilled eccentrically, then the hole drilling experiment, as formulated by E 837 standard, cannot be used for any more complex determination of the strain state in the vicinity of the drilled hole, which would be necessary for any eventual improving corrections. This simple standard drilled theory is not probably reliable for imperfections occurring in drilled holes. In [4, 5], we describe the principle used for an objective experimental evaluation of surface strains, where all stress components are modified by the seven parameters as described in Eq. (4). There we do not have to use the completely rosette strain gages around the drilling hole but only short winding segments or winding points of this strain gages. We expect that stress state components in the surroundings of the blind drilled hole, as written in Eq. (4), are analogous to those by Eq. (3) used for a normal straight-through hole. Let we also modify all the seven terms of the complete Kirsch's theory by parameters $c_k(r,\alpha,z)$, which are dependent on the distance r from the center of the drilled hole, angle α from a direction related to the principal stress σ_x and the depth z of the blind drilled hole. By the way, a similar approach is also used by E 837 standard for strain - gage strains. These complete components of the stress state change, induced by drilling the hole, can be transformed to the strain components. A strain state on planes perpendicular to the surface can be set by an angular transformation, where the use of the first three components (see Fig. 1) $\varepsilon_r, \varepsilon_\Theta, \gamma_{r\Theta}$ in Eq. (7) is sufficient, because the principal strain ε_z does not have any effect on it. Fig. 2 defines the position of the

$$\varepsilon_g = \frac{\varepsilon_\Theta + \varepsilon_r}{2} + \frac{\varepsilon_\Theta - \varepsilon_r}{2}\cos 2\varphi + \frac{\gamma_{\Theta,r}}{2}\sin 2\varphi \qquad (6)$$

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\Theta \\ \gamma_{r\Theta} \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_r \\ \sigma_\Theta \\ \tau_{r\Theta} \end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix} \left\{\left[-\frac{c_1}{r^2} - \frac{c_4}{r^2}\nu\right] + \left[\frac{c_2}{r^4}3 + \frac{c_5}{r^4}3\nu - \frac{c_3}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\frac{c_4}{r^2} + \frac{c_1}{r^2}\nu\right] - \left[\frac{c_5}{r^4}3 + \frac{c_2}{r^4}3\nu - \frac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\frac{c_6}{r^4}3 + \frac{c_7}{r^2}2 - \frac{c_6}{r^4}3\nu + \frac{c_7}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\frac{c_1}{r^2}\nu - \frac{c_4}{r^2}\nu\right] + \left[-\frac{c_2}{r^4}3\nu + \frac{c_5}{r^4}3\nu + \frac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \qquad (7)$$

wind g axis towards Θ axis for an acute angle φ. The strain in the g direction is derived from $\varepsilon_r, \varepsilon_\Theta, \gamma r_\Theta$ strains according to the Mohr's transformation Eq. (6) by the use of goniometric functions of a double angle 2φ.

2. Regression model

Parameters $c_1, \dots c_7$ in Eq. (4, 7) of analytical model representing a hole drilling method can be determined via regression of the results yield by FEM. The numerical FEM experiment models a flat test beam with a rectangular cross-section. The specimen is loaded with a unidirectional principal stress σ_x collinear with beam axis of symmetry. Further, the same specimen with a drilled hole produced by a drilling process is modeled by FEM as well, where the hole is normal to the test surface at the beam axis of symmetry. In order to compute unknown parameters of the model, it is possible to use either simpler stress equations Eq. (4) or derived the strain equations according Eq. (7). The assumed regression functions in Eq. (4) are goniometrical, analogous to Kirsch's theory stated in Eq. (2). Based on the experience with the regression model conforming to the E837 standard for drilling method as stated in Eq. (5), it is not required to assume the dependency of the identified parameters on the relative position to the hole expressed by angle α. The parameters $c_k(r,z)$ can be therefore identified with the data yielded by a numerical model of drilling in the surface layer of the first hole quadrant mapped with coordinates x, y as depicted in Fig. 1. These are therefore dependent only on the radial distance from the center of the drilled hole r and on the hole depth z.

$$\left\{\begin{aligned}
&\min\sum_i(\sigma_r-\bar{\sigma}_r)_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\Big(-\frac{1\cdot c_1(r,z)}{r^2}\Big)+\frac{\sigma_x}{2}\Big(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\Big)\cos 2\alpha-\bar{\sigma}_r\right]_i^2=\min F1(c_1,c_2,c_3)\\
&\min\sum_i(\sigma_\theta-\bar{\sigma}_\theta)_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\Big(\frac{1\cdot c_4(r,z)}{r^2}\Big)-\frac{\sigma_x}{2}\Big(\frac{3\cdot c_5(r,z)}{r^4}\Big)\cos 2\alpha-\bar{\sigma}_\theta\right]_i^2=\min F2(c_4,c_5)\\
&\min\sum_i(\tau_{r\theta}-\bar{\tau}_{r\theta})_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\Big(-\frac{3\cdot c_6(r,z)}{r^4}+\frac{2\cdot c_7(r,z)}{r^2}\Big)\sin 2\alpha-\bar{\tau}_{r\theta}\right]_i^2=\min F3(c_6,c_7)
\end{aligned}\right\} \tag{8}$$

$$\left\{\begin{aligned}
&\frac{\partial}{\partial c_1}\sum_i(\sigma_r-\bar{\sigma}_r)_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(-\frac{1\cdot c_1(r,z)}{r^2}\Big)+\frac{\sigma_x}{2}\Big(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\Big)\cos 2\alpha-\bar{\sigma}_r\right\}\cdot\Big(-\frac{\sigma_x}{2r^2}\Big)\right]_i=0\\
&\frac{\partial}{\partial c_2}\sum_i(\sigma_r-\bar{\sigma}_r)_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(-\frac{1\cdot c_1(r,z)}{r^2}\Big)+\frac{\sigma_x}{2}\Big(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\Big)\cos 2\alpha-\bar{\sigma}_r\right\}\cdot\Big(\frac{3\sigma_x}{2r^4}\cos 2\alpha\Big)\right]_i=0\\
&\frac{\partial}{\partial c_3}\sum_i(\sigma_r-\bar{\sigma}_r)_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(-\frac{1\cdot c_1(r,z)}{r^2}\Big)+\frac{\sigma_x}{2}\Big(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\Big)\cos 2\alpha-\bar{\sigma}_r\right\}\cdot\Big(-\frac{2\sigma_x}{r^2}\cos 2\alpha\Big)\right]_i=0\\
&\frac{\partial}{\partial c_4}\sum_i(\sigma_\theta-\bar{\sigma}_\theta)_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(\frac{1\cdot c_4(r,z)}{r^2}\Big)-\frac{\sigma_x}{2}\Big(\frac{3\cdot c_5(r,z)}{r^4}\Big)\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\Big(\frac{\sigma_x}{2r^2}\Big)\right]_i=0\\
&\frac{\partial}{\partial c_5}\sum_i(\sigma_\theta-\bar{\sigma}_\theta)_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(\frac{1\cdot c_4(r,z)}{r^2}\Big)-\frac{\sigma_x}{2}\Big(\frac{3\cdot c_5(r,z)}{r^4}\Big)\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\Big(-\frac{3\sigma_x}{2r^4}\cos 2\alpha\Big)\right]_i=0\\
&\frac{\partial}{\partial c_6}\sum_i(\tau_{r\theta}-\bar{\tau}_{r\theta})_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(-\frac{3\cdot c_6(r,z)}{r^4}+\frac{2\cdot c_7(r,z)}{r^2}\Big)\sin 2\alpha-\bar{\tau}_{r\theta}\right\}\cdot\Big(-\frac{3\sigma_x}{2r^4}\sin 2\alpha\Big)\right]_i=0\\
&\frac{\partial}{\partial c_7}\sum_i(\tau_{r\theta}-\bar{\tau}_{r\theta})_i^2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\Big(-\frac{3\cdot c_6(r,z)}{r^4}+\frac{2\cdot c_7(r,z)}{r^2}\Big)\sin 2\alpha-\bar{\tau}_{r\theta}\right\}\cdot\Big(\frac{\sigma_x}{r^2}\sin 2\alpha\Big)\right]_i=0
\end{aligned}\right\} \tag{9}$$

The mesh of identification nodes, mapping the first quadrant of the neighborhood of the drilled hole, is performed in polar coordinates r and α. The region of radii used for identification is assumed according a possible sensitivity of the drilling experiment in interval

$1 < r_j < 5$, which is further split into 21 levels with step 0,2 and with angle coordinates α split in interval $0 < \alpha_k < \pi/2$ into 10 steps with size of $\pi/20$. This variable sampling induces a field of identification points, which can used in Eq.(4). This equation for σ_r, σ_Θ, $\tau_{r\Theta}$ must in be in all nodes satisfied. Possible degrees of freedom, unknown parameters $c_1,\dots c_7$ can be determined with use of least squares method, which minimizes residual errors between the analytical and numerical methods. This task can be transformed into minimization of three independent functionals *F1, F2* and *F3*, yielding seven linear equation system in form of Eq. (9).

The conditions for the minimization of the functionals *F1, F2* and *F3* can be separated into three independent linear equation systems as stated in Eq. (10). The first three equations follow from *F1*, other two from *F2* and finally the sixth and seventh equations from *F3*. Unknown parameters $c_1,\dots c_7$ can be determined from many possible initial point combinations, which can be selected from many various radii and depths. The most important criterion for the point selection identification is capability of analytical model to regress the measured values. For example, a relative drill depth $z/2R_0 = 0.025$ and a radius range between $1.2 \le r_j \le 5$ with a step of 0.2 induces 200 identification points, which can be decomposed into 600 points for three strains, which can be incorporated into Eq. (10). In this case, the number of points for the regress procedure is too high as the analytical model Eq. (4) does not fit the measured values properly – en error in some points can be in tens of percent. Convenient set of parameters can be found from points measured at one particular radius at angles between $0 \le \alpha_k \le \pi/2$.

$$
\left.
\begin{aligned}
&\begin{bmatrix}
\sum\limits_i -\dfrac{1}{r^4} & \sum\limits_i \dfrac{3\cos 2\alpha}{r^6} & -\sum\limits_i \dfrac{4\cos 2\alpha}{r^4} \\
\sum\limits_i -\dfrac{\cos 2\alpha}{r^6} & \sum\limits_i \dfrac{3\cos^2 2\alpha}{r^8} & -\sum\limits_i \dfrac{4\cos^2 2\alpha}{r^6} \\
\sum\limits_i -\dfrac{\cos 2\alpha}{r^4} & \sum\limits_i \dfrac{3\cos^2 2\alpha}{r^6} & -\sum\limits_i \dfrac{4\cos^2 2\alpha}{r^4}
\end{bmatrix}
\cdot
\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}
= \frac{2}{\sigma_x} \cdot
\begin{Bmatrix}
\sum\limits_i \dfrac{\overline{\sigma}_r}{r^2} \\
\sum\limits_i \dfrac{\overline{\sigma}_r}{r^4} \cos 2\alpha) \\
\sum\limits_i \dfrac{\overline{\sigma}_r}{r^2} \cos 2\alpha)
\end{Bmatrix} \\
&\begin{bmatrix}
\sum\limits_i \dfrac{1}{r^4} & \sum\limits_i -\dfrac{3\cos 2\alpha}{r^6} \\
\sum\limits_i \dfrac{\cos 2\alpha}{r^6} & \sum\limits_i -\dfrac{3\cos^2 2\alpha}{r^8}
\end{bmatrix}
\cdot
\begin{Bmatrix} c_4 \\ c_5 \end{Bmatrix}
= \frac{2}{\sigma_x} \cdot
\begin{Bmatrix}
\sum\limits_i \dfrac{\overline{\sigma}_\theta}{r^2} \\
\sum\limits_i \dfrac{\overline{\sigma}_\theta}{r^4} \cos 2\alpha
\end{Bmatrix} \\
&\begin{bmatrix}
\sum\limits_i -\dfrac{3\sin^2 2\alpha}{r^8} & \sum\limits_i \dfrac{2\sin^2 2\alpha}{r^6} \\
\sum\limits_i -\dfrac{3\sin^2 2\alpha}{r^6} & \sum\limits_i \dfrac{2\sin^2 2\alpha}{r^4}
\end{bmatrix}
\cdot
\begin{Bmatrix} c_6 \\ c_7 \end{Bmatrix}
= \frac{2}{\sigma_x} \cdot
\begin{Bmatrix}
\sum\limits_i \dfrac{\overline{\tau}_{r\theta}}{r^4} \sin 2\alpha \\
\sum\limits_i \dfrac{\overline{\tau}_{r\theta}}{r^2} \sin 2\alpha
\end{Bmatrix}
\end{aligned}
\right\}
\tag{10}
$$

3. Conclusions

The issue of the regression models determination is analogous with ensuing theories, which further expands a field of application of this method. These theories are: i) the theory of stress state identification after the hole drilling method application [7], and ii) the theory of sensitivity gain of the hole drilling method for the stress state identification [6]. Further increase of the drilling method sensitivity by a hole enlargement is limited. In the neighborhood of the hole between $1 \leq r_j \leq 1.5$, there is a high gradient of the strains ε_r, ε_Θ, γr_Θ, as depicted in Fig. 3, where dramatic changes in the relative radial deformation for selected angles α can be noted. The deformations, measured with strain-gauges, in the neighborhood of the drilled hole, are comparable with those of the relative radius change of $\Delta r = 0.5$. In the case of changes in the strain signs, which can locally occur, the drilling method has not to be convenient for the stress identification.

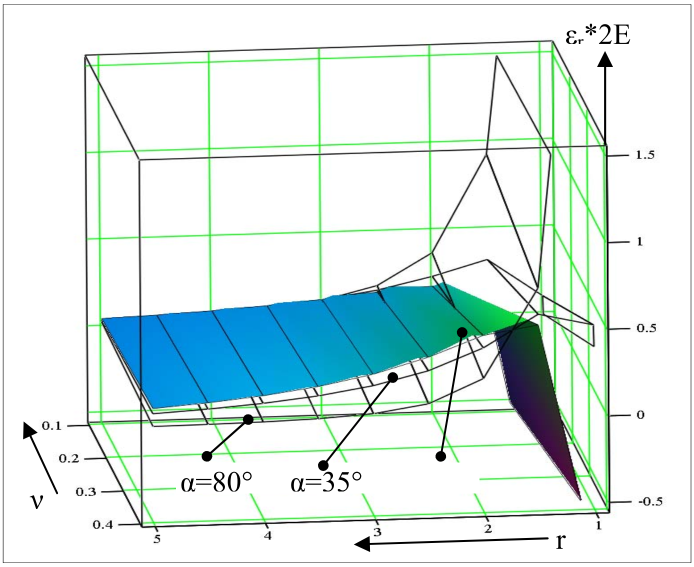

Fig. 3. Sensitivity of strain ε_r on Poisson's ratio $\boldsymbol{\nu}$ and distance from the drilled hole $\boldsymbol{r}$

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