

## Measurement of thermal expansion coefficients of unidirectional composites using strain gages

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**Abstract:** The article focuses on a determination of a coefficient of thermal expansion (CTE) of unidirectional thick-walled carbon composite specimens. The CTE were determined using two measurement methods. Both methods are based on a measurement of thermal output of resistance strain gages, installed on a stress-free specimen, which is subjected to the temperature change. The first method uses single strain gage, which is bonded to a specimen made of test material. Value of CTE is refined from measured thermal output using corrective equations. The second method uses a pair of strain gages, with one bonded to a specimen from a reference material, and the second to a specimen from the tested material. The specimen's CTE can be then calculated from the difference in expansion coefficients, which is equal to the unit difference in thermal output. Measured values of CTE were compared with values calculated analytically.

**Keywords:** coefficient of thermal expansion; CTE; strain gage

### 1. Introduction

Fibre composites has been widely used in the last decades in aerospace industry as a light-weight structural material that allows to design planes with a lower mass, lower fuel consumption, improved fatigue resistance, extended time between maintenance controls, etc. The application of fibre composites nowadays, while still growing in aerospace industry has also spread to other areas like automotive, sports and also to industrial applications. One of the fibre composites industrial applications is their usage in components of machine tools. However, these applications have completely different demands and motivation than the applications of fibre composites in means of transport.

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Machine tool's design demands components with excellent static stiffness (loading: bending, torsion, transverse shear), modal properties and also with dimensional stability. Bending stiffness together with modal properties can be improved by application of high or ultra-high-modulus carbon fibres. These materials with modulus in direction of fibres around 600 or 800 GPa have a huge potential for improvement of machine tools, however due to other requirements (connection interfaces, transverse shear stiffness, price of high-performance fibres) a presence of steel in a design is necessary.

Combination of steel/carbon composites can provide a functional design, however it is necessary to develop the suitable design knowledge to prevent problems with strength and dimensional stability due to a different thermal behaviour of each material. Another motivation for a good design tool for prediction of thermal behaviour is a possibility of improving of machine tool precision by fibre composites with almost zero thermal expansion in a given direction as this application can significantly improve a dimensional stability in machining conditions. The most important parameter for a description of the aforementioned behaviour is a coefficient of thermal expansion (CTE).

This work focuses on a determination of CTE of composite specimens that are suitable for application in a machine tool design. Due to the significant requirements on static stiffness, the components tend to be designed as thick-walled structures and have to be manufactured by specific manufacturing technologies (combination of filament winding, fibre placement, pressing). Combination of complicated manufacturing technologies (and their repeatability) together with a thick-walled design leads to a difficult determination of CTE by analytical tools which were derived for classic composites that are used in aerospace. This work focuses on experimental determination of CTE of thick-walled composite specimens using resistance strain gages. As the CTE of composite materials is anisotropic, focus was held on a suitable thermal correction of measured values.

## 2. Definition of experimental specimens

Specimens for experimental determination of CTE were made from carbon fibres and epoxy resin in a shape of a prism, with dimensions of 20 x 30 mm (cross-section) and length from 35 mm to 250 mm. Three types of carbon fibres: 1 type of high-strength carbon fibres and two types of ultra-high modulus graphite fibres were used. Nominal properties of the fibres and matrix are given in Table 1.

**Table 1. Nominal properties of materials used in experimental specimens**

	$\rho_L$ [kg.m <sup>-3</sup> ]	$E_L$ [GPa]	$E_T$ [GPa]	$G_{LT}$ [GPa]	$\nu_{LT}$ [-]	$\alpha_L$ [K <sup>-1</sup> ]	Source
T700	1800	230	15	50	0.30	-0.38E-06	[1]
k63712	2120	640	6	10	0.20	-1.38E-06	[2], [3]
CN80	2170	780	5	20	0.35	-1.5E-06	[4]
epoxy	1200	3÷4	3÷4	1.6	0.4	40÷60E-06	[5]

Although the values in direction of fibres are often published by fibres suppliers (see Table 1), the values of transverse properties are published rarely. Approximate values of carbon fibres transverse thermal expansion can be found in [6] where the transverse CTE is given in range  $9\div 12.5E-6\text{ K}^{-1}$  or in [7] where transverse CTE is given for high-strength, high and ultra-high modulus carbon fibres in range between  $7\div 10E-6\text{ K}^{-1}$ .

Experimental specimens were prepared by CompoTech Plus, s.r.o. Nominal mechanical constants are given in Table 2. Values given in Table 2 were obtained using mixed rules for classic composite materials from determined fibre volume fraction and nominal constants given in Table 1. Experimental specimens are shown in Fig. 1 and Fig. 2.

**Table 2. Nominal mechanical properties of experimental specimens**

	$V_f$ [%]	$\rho_L$ [kg.m <sup>-3</sup> ]	$E_L$ [GPa]	$E_T$ [GPa]	$G_{LT}$ [GPa]	$\nu_{LT}$ [-]
T700/E	46	1450	108	7.3	2.97	0.35
k63712/E	59	1723	380	9.7	3.03	0.38
CN80/E	50	1657	380	7.9	3.26	0.28



**Fig. 1.** Cross-section of thick-walled UD carbon composite specimens.



**Fig. 2.** Unidirectional specimen from CN80/E composite with applied strain gage rosettes.

### 3. Description of methods for determination of CTE

Three methods were used for determination of longitudinal and transverse CTE of the experimental specimens:

- 1) Analytical approach
- 2) Direct CTE measurement technique
- 3) Reference CTE measurement technique

#### 3.1. Analytical solution of CTE

Value of CTE in longitudinal direction of unidirectional composite, i.e. in directions of fibres can be determined by equation (1) which was derived from an energy balance by Sharperry, see [8]. In the same work, equation (2) that determines a value of CTE in transverse direction was derived. However, equation (2) is valid only for isotropic fibres. Therefore, it should not be used for anisotropic carbon fibres.

$$\alpha_1 = \frac{\alpha_{Lf} \cdot E_{Lf} \cdot V_f + \alpha_m \cdot E_m \cdot V_m}{E_1} \quad (1)$$

$$\alpha_2 = (1 + \nu_m) \cdot \alpha_m \cdot V_m + (1 + \nu_{12f}) \cdot \alpha_{fL} \cdot V_f - \alpha_1 \nu_{12} \quad (2)$$

According to [7], the most accurate values of CTE in transverse direction can be determined from Rosen and Hashin model [9]. However, this model is computationally complicated and is not applicable in common engineering calculations. CTE in transverse direction is influenced mostly by behaviour of matrix with minor influence of fibres transverse expansion. For determination of this value, a less accurate equation (3) in comparison with models [9] was used. This equation was derived by Strife and Prewo [10] from equation (2) using a transverse value of CTE  $\alpha_{fT}$  of fibres instead of longitudinal value of CTE  $\alpha_{fL}$ .

$$\alpha_2 = (1 + \nu_m) \cdot \alpha_m \cdot V_m + (1 + \nu_{12f}) \cdot \alpha_{fT} \cdot V_f - \alpha_1 \nu_{12} \quad (3)$$

Both longitudinal and transverse values of CTE of the experimental specimens were determined from equation (1) and (3) using values given in Table 1 and Table 2. CTE of fibres in transverse direction was taken as  $10\text{E-}06 \text{ K}^{-1}$  for all types of fibres; CTE of matrix was taken as  $45\text{E-}06 \text{ K}^{-1}$ .

### 3.2. Direct CTE measurement technique

Method of direct measurement of CTE is based on a strain gage measurements on the surface of the investigated material. While the specimen from the tested material is exposed to a temperature change, strain from strain gage and temperature from thermometer are measured. The actual value of CTE is then evaluated using the corrective equations. This technique requires the installation of one strain gage (in the case of isotropic material) or a strain gauge rosette (in the case of measurement on the composite, or other anisotropic material). Temperature sensor must be placed on the specimen surface as close as possible to the strain gage.

Relative deformation measured by the strain gage is described by the following equation:

$$\epsilon_{ind} = \left[ \frac{\beta_G}{F_G} + (\alpha_S - \alpha_G) \right] \cdot \Delta T \quad (4)$$

- where:  $\beta_G$  ... thermal coefficient of resistivity of a grid material  
 $\alpha_S - \alpha_G$  ... difference in CTE of a specimen and grid  
 $F_G$  ... gage factor  
 $\Delta T$  ... change in the temperature

Pure indicated strain  $\epsilon_{ind}$  needs corrections, including correction for transverse sensitivity, especially in case of strain gage rosettes (equations are given by SG manufacturer) and corrections for temperature dependent behaviour of the strain gage factor. Temperature dependent relative deformation of a surface of the tested specimen  $\epsilon_{mat}$  is then evaluated by equation (5).

$$\varepsilon_{\text{mat}} = \varepsilon_{\text{ind-corr}} + \varepsilon_G - \varepsilon_{G\text{-app}} \quad (5)$$

where:  $\varepsilon_{\text{ind-corr}}$  ... indicated strain after corrections  
 $\varepsilon_G$  ... thermal deformation of a grid  
 $\varepsilon_{G\text{-app}}$  ... apparent strain

Finally, CTE is evaluated using a linear interpolation of the relative deformation  $\varepsilon_{\text{mat}}$  depending on the change in measured temperature. The calculated coefficients of thermal expansion must be further converted into the main deformation directions - longitudinal  $\varepsilon_1$  ( $\alpha_1$ ) and transverse  $\varepsilon_2$  ( $\alpha_2$ ).

Direct measurement technique was evaluated, before application on composite specimens, by a measurement of CTE of copper (isotropic material). The measured value of CTE is  $16.8\text{E-}06 \text{ K}^{-1}$ , which is in agreement with tabular value ( $16.5 - 17.0\text{E-}06 \text{ K}^{-1}$ ).

### 3.3. Reference CTE measurement technique

The reference method for determining the CTE of materials is based on the typical behaviour of the strain gage response to the change of CTE of material. The curve, describing the relationship between the  $\Delta\varepsilon$  and  $\Delta T$ , keeps the same shape for different materials (and CTEs), but it rotates. For materials with higher CTE curve rotates anti-clockwise; for materials with lower CTE curve rotates clockwise. The rotation of the curve is caused only by changes in the CTE value (for example when the strain gages of a same type are installed on different materials). Considering this, a relationship that describes temperature dependent strain gage behaviour (4) can be modified in a following way:

Equation (6) describes the thermal response of the strain gage (in the form of relative deformation) that is installed on a specimen, whose CTE  $\alpha_S$  is measured. Equation (7) expresses the thermal response of the strain gauge installed on the surface of a material with known CTE  $\alpha_R$  (reference specimen).

$$\varepsilon_S = \left[ \frac{\beta_G}{F_G} + (\alpha_S - \alpha_G) \right] \cdot \Delta T \quad (6)$$

$$\varepsilon_R = \left[ \frac{\beta_G}{F_G} + (\alpha_R - \alpha_G) \right] \cdot \Delta T \quad (7)$$

By subtracting (6) – (7) and rearranging the required  $\alpha_S$  is expressed:

$$\alpha_S = \frac{\varepsilon_S}{\Delta T} - \frac{\varepsilon_R}{\Delta T} + \alpha_R \quad (8)$$

The method works directly with the measured data ( $\varepsilon_{\text{ind}}$ ). Corrections are not necessary (except of corrections for rosettes), as is evident from equation (8). The influence of the strain gage grid material, adhesives for the installation of strain gages, wiring, etc. does not enter into the calculation (provided that the installation procedure was the same for all strain gages, the cables have the same resistance, etc.). Accuracy of the method is directly dependent on the accuracy of CTE determination of the reference material. Measurement error due to transverse sensitivity of strain gauge remains, as in case of direct measurement technique. Finally, the calculated CTEs are recalculated into the main deformation directions.

Specimen made of Invar was used as a reference material. Its CTE was evaluated by three independent methods: *direct measurement technique* ( $2.95\text{E-}06 \text{ K}^{-1}$ ), *contact displacement sensor* ( $3.21\text{E-}06 \text{ K}^{-1}$ ), *contactless displacement sensor* ( $3.01\text{E-}06 \text{ K}^{-1}$ ). Average value of CTE from all three methods is entering equation (8).

## 4. Experimental setup and results

### 4.1. Experimental setup

HBM 1-RY81-6/350 strain gage rosettes were bonded to the surface of the specimens using the HBM X60 fast curing two-component adhesive. Standard strain gage installation process (recommended by the manufacturer for the composites) was used. Strain gages were coated with transparent silicon rubber HBM SG250 and with cellulose pads to avoid the impact of ambient temperature. Analogical installation process was used for the Pt100 resistive thermometers. HBM Spider8 multi-channel electronic PC measurement unit was used to capture data from the strain gages and thermometer. Excitation voltage was set to 2.5 V to eliminate thermal drift. Shielded copper cables of the same length and resistance were used to connect strain gages and Spider8 (three-wire quarter-bridge strain gage circuit). Specimen configuration and detail of strain gage rosette installation are shown in Fig. 3 and 4.



**Fig. 3.** Specimen made of unidirectional Carbon/Epoxy composite (Dialead k63712).



**Fig. 4.** Detail of HBM 1-RY81-6/350 strain gage rosette installation.

Specimens were heated in electrical laboratory furnace with heating rate set to  $0.5 \text{ }^\circ\text{C}$  per minute in the temperature range  $20 \text{ }^\circ\text{C}$  to  $60 \text{ }^\circ\text{C}$ . Both heating and cooling cycles were measured. So-called "training cycles" were carried out to ensure proper curing of the adhesive and accuracy of measurements.

### 4.2. Experimental results and discussion

Values of CTE in main deformation directions (1 – longitudinal direction; 2 – transverse direction) are given in Table 3 together with results of analytical models. The values of CTE of experimental specimens that were obtained by the direct measurement technique match with only small variations the values that were measured by the reference measurement technique.

**Table 3. A comparison of measured and calculated values of CTE**

		direct method	reference method	analytical solution
<b>T700/E (UD)</b>	$\alpha_1$ [K <sup>-1</sup> ]	3.32E-06	3.43E-06	0.27E-06
	$\alpha_2$ [K <sup>-1</sup> ]	40.21E-06	40.32E-06	37.60E-06
<b>k63712/E (UD)</b>	$\alpha_1$ [K <sup>-1</sup> ]	2.01E-06	2.14E-06	-1.19E-06
	$\alpha_2$ [K <sup>-1</sup> ]	41.28E-06	41.41E-06	31.5E-06
<b>CN80/E (UD)</b>	$\alpha_1$ [K <sup>-1</sup> ]	0.07E-06	0.19E-06	-1.27E-06
	$\alpha_2$ [K <sup>-1</sup> ]	47.66E-06	47.78E-06	36.80E-06

A significant difference between the CTE obtained from the experimental work and analytical solutions can be seen in Table 3. Theoretically, this difference can be caused either by a non-correct evaluation of CTE from experimental values or by a non-correct analytical solution or by combination of the both factors.

Although the two presented experimental methods lead to similar values of CTE, both methods are influenced by transverse sensitivity of strain gages. Strain gages transverse sensitivity have been designed and calibrated for measurements on isotropic materials, therefore strain gage corrections for transverse sensitivity are known for isotropic materials only.

Analytical solution is influenced by accuracy of material constants (CTE of fibres and matrix) and parameters from specimens manufacturing (fibre volume fraction, presence of voids, through-thickness quality). All the material constants were taken from fibres and matrix suppliers, or even from a literature for similar materials, therefore the computational error can be significant (even without including technological parameters).

Sensitivity analysis of material constants that are entering analytical solutions was performed. Values of fibres CTE were evaluated so that the analytical solution of  $\alpha_1$  parameters fits the values from the performed experiments. Results of the analysis are following:

- 1) CTE of T700 fibres: nominal: -0.38E-06 K<sup>-1</sup>; fitting value: 2.45E-06 K<sup>-1</sup>
- 2) CTE of k63712 fibres: nominal: -1.38E-06 K<sup>-1</sup>; fitting value: 1.90E-06 K<sup>-1</sup>
- 3) CTE of CN80 fibres: nominal: -1.50E-06 K<sup>-1</sup>; fitting value:-1.50E-07 K<sup>-1</sup>

This comparison confirms that not only the analytical solution is influenced by the aforementioned errors, but also the experimental methods were influenced by a transverse sensitivity of strain gages. The main reason is that the material constants of carbon fibres would have to be significantly different from their real physical values.

## 5. Conclusions

Two CTE measurement methods, based on strain gages, were performed and compared with results of analytical solution. Both measurement techniques provide practically identical results. A significant difference was observed between the measured and analytical CTE values. These differences may result from influence of transverse sensitivity of strain gages and accuracy of constants entering the analytical solution. Further research must be focused on determination of relationship between the change in strain gage signal and location of sensor on UD composite specimen (angle between the fibres and the longitudinal axis of strain gage). Refinement of material constants is necessary. For example, by measurement of CTE of matrix without fibres or with various fibre volume fractions.

## Acknowledgements

"This work was supported by the Grant Agency of the Czech Technical University in Prague, grant No. SGS10/261/OHK2/3T/1". Authors would like to acknowledge the support by Technology Agency of the Czech Republic in project TA02010543.

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