EAN 2012
50th annual coference on experimental stress analysis

Experimental identification of relaxation parameter from relaxation test of composite tube

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Abstract: This paper is focused on the relaxation inflation tests of a composite tube manufactured from three layers. The first layer is formed from a thin latex tube, the second one, that increases a stiffness of specimen, is formed from a helically wounded textile rubber band and the third one is elastic matrix. Elastic matrix connects the first layer with the second one also.

Experiments are based upon evaluation of recorded pressure responses to the stepwise increase of water volume inside the inflated pipe. The aim of experiments is to identify a relaxation parameter of a suggested viscoelastic constitutive equation, which is based upon the Haslach's principle of maximum dissipated energy during transition from a non-equilibrium towards the equilibrium state. Results obtained from simulations were compared with experimental measurements carried out with a viscoelastic tube and relaxation parameter was obtained.

Keywords: Viscoelasticity; Relaxation; Composite tube

1. Introduction

Viscoelastic processes are non-equilibrium time dependent processes. Some energy is reversibly stored during loading and some is dissipated to heat. Several approaches exist for viscoelastic behaviour description of solids.

One approach is utilization of the hereditary integral formulation based on Boltzmann superposition principle for modelling nonlinear viscoelastic behaviour developed by Coleman and Noll [3] and used for soft tissue by Fung [4] who named this approach as Quasi-linear viscoelasticity (QLV). Many researchers adopted and adapted QLV theory to fit the responses of soft tissues Abramowitch and Woo [1], Funk et. al. [5], Lynch et. al. [13], Sarver et. al. [14], Toms et. al. [15], Valdez-Jasso [17], Craiem [18]. There are also phenomenological models that are derived from parallel or serial connection of elastic springs and viscous dampers Valdez-Jasso [16], Bessems [2]. Other approach, Holzapfel [10, 11], describes viscoelastic processes by evolution equations of inelastic strains or stresses, however the foundations of this approach are still derived from phenomenological models of connected springs and dashpots. The transversely isotropic viscohyperelastic

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material introduced in the Limbert and Middleton model [12] is based on a definition of a general Helmholtz free energy function which is a sum of hyperelastic and viscous potential. Their approach is capable to describe anisotropic viscous behaviour also. Haslach [6- 9] introduced a new class of non-equilibrium thermoviscoelastic evolution equations based on long-term behaviour and a maximum dissipation principle for polymers, rubbers and soft tissues. The non-linear evolution equations (1) for thermoviscoelastic behaviour in terms of state variables, x_i, and control variables, y_i, are generated from long-term constitutive models represented by an energy function Ψ used for elasticity, see below.

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \cdots \\ \frac{dx_n}{dt} \end{bmatrix} = -k \begin{bmatrix} \frac{\partial^2\psi}{\partial x_1^2} & \frac{\partial^2\psi}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2\psi}{\partial x_1 \partial x_n} \\ \frac{\partial^2\psi}{\partial x_2 \partial x_1} & \frac{\partial^2\psi}{\partial x_2^2} & \cdots & \frac{\partial^2\psi}{\partial x_2 \partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2\psi}{\partial x_n \partial x_1} & \frac{\partial^2\psi}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2\psi}{\partial x_n^2} \end{bmatrix}^{-2} \begin{bmatrix} -y_1(t) + \frac{\partial\psi}{\partial x_1} \\ -y_1(t) + \frac{\partial\psi}{\partial x_1} \\ \cdots \\ -y_n(t) + \frac{\partial\psi}{\partial x_n} \end{bmatrix} \tag{1}$$

This approach reduces the number of experiments and there is no need to obtain creep or relaxation function for description of these phenomena. Moreover, the classical spring and dashpot linear models were recovered from this hypothesis (Kelvin-Voight model, Standard Linear Solid model).

This paper deals with a relaxation experiment on a composite tube. Only one specimen of elastic tubes was tested: a simple tube having a composite three layers structure. The primary aim is to obtain a relaxation parameter of the constitutive model for description of the tested specimen during relaxation test. Haslach's construction of thermostatic nonlinear evolution equation was utilized for this purpose. The investigated parameter is relaxation parameter.

2. Methods

2.1. Manufacture of composite tube

Composite tube represents a physical model of blood vessel. Physical model was developed as tube with three layers. The first (intima) layer was formed from a thin wall latex tube. The second layer (media) was formed from rubber band helically wounded on the outer surface of the first layer. The rubber band increases its stiffness significantly when a large deformation is achieved (tested bands have the limiting stretch ratio 2). The connection between the first and the second layer was realized by a silicone matrix. Silicone matrix formed also the third layer.

2.2. Experiment - Inflation test

Inflation test was carried out to obtain the dependence between pressure and volume, i.e. to provide information on purely elastic behaviour of the tested specimen at equilibrium state. The physical model was inflated by a small predefined increment of volume and the corresponding pressure was recorded by a pressure transducer.

2.3. Experiment – Relaxation test

The pressure time-dependent characteristic was obtained from relaxation test. Relaxation experiment proceeds this way: at the beginning the tested specimen was in equilibrium state with inner zero overpressure, then the physical model was almost instantaneously pressurized by defined increase of the volume with aid of syringe. The pressure transducer started record pressure at the same time when the tested specimen was inflated. Experimental setup scheme is showed bellow Fig. 1.

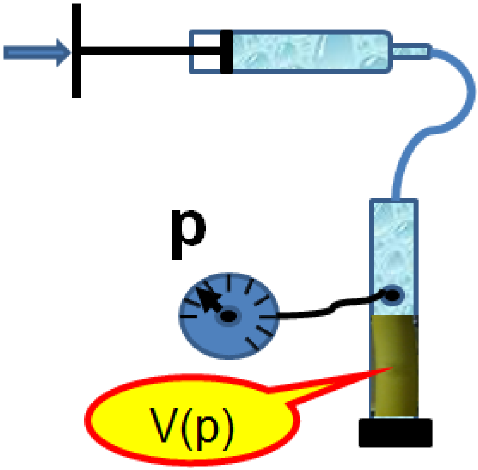

Fig. 1. Relaxation experiment setup

The tested specimen had three degrees of freedom (radial, circumferential and axial direction).

The dimensions of the tested specimen are presented in Table 1.

Table 1. The dimensions of the physical model

Dimension	Size (mm)
Inner radius R_0	8.65
Wall thickness H_0	1.86
Length of the specimen L_0	54.22

2.4. Mathematical model

Mathematical model is derived from system of equations (1). The simplification of the mathematical model is based on assumption that pressure is constant in entire water volume of the tested specimen. Therefore, the system of evolution equations

(1) is reduced to one equation (2) where the state variable *x*, correspond to pressure, *p*, and the control variable, *y*, coincides to the specimen volume *V*.

$$\frac{dp}{dt} = -k\left(\frac{\partial^2\psi}{\partial p^2}\right)^{-2}\left(-V + \frac{\partial\psi}{\partial p}\right) \tag{2}$$

Constant *k* represents relaxation parameter in (2). Now, it is necessary to define a density energy function *Ψ*. In the case of stress relaxation test, it is suitable to define the density energy function as the complementary energy by equation (3).

$$\psi = \int_0^p V dp \tag{3}$$

The differential equation (2) is solved by means of implicit Euler method. Initial values are obtained from experimental measurement. Volume *V* in (2) is fixed during the experiment.

3. Results

3.1. Elastic response

The inflation test of the blood vessel physical model revealed nonlinear volume-volume-pressure relationship, see Fig. 2.

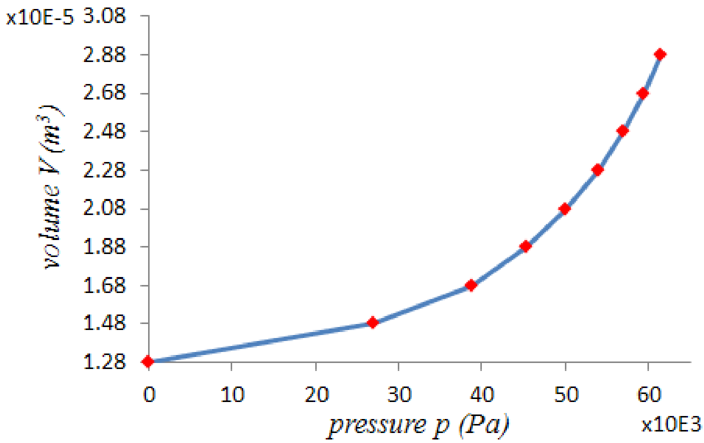

Fig. 2. Inflation test results

The model (linear spline model) was adopted for description of the pure elastic behaviour. The model fit experimental data successfully.

3.2. Time-dependent response

Results from simulation and from relaxation test are shown in Fig. 3. Model is represented by blue line and red markers correspond to the experimental data. In order to achieve the appropriate relaxation the relaxation coefficient *k* was set to 6E-11 $m^3Pa^{-1}s^{-1}$.

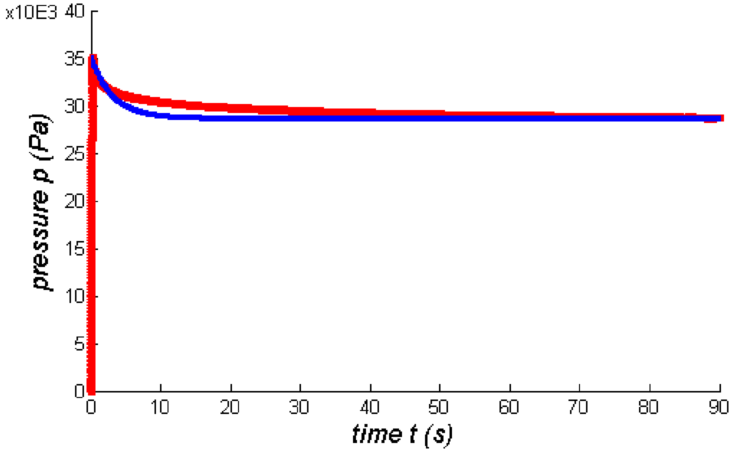

Fig. 3. Relaxation test results

4. Conclusion

Inflation test data was fitted successfully. The one parameter model utilized for simulation of relaxation test is not with good agreement with experimental data from measurement. The inaccuracy of the simulation may be due to the fact that the model was designed as an isotropic 0D model. Therefore, it is possible that simplifying assumptions for axial and radial stretches (axial and radial stretches are assumed to be zero) and neglect of anisotropy leads to inaccurate approximation of the model to experimental data.

Acknowledgements

This work has been supported by SGS ČVUT 2010 OHK2-005/10 - Research of technical and biological composite materials (2010-2012).

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