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Bulge Test of Composite Membrane with Elastomeric Matrix Reinforced by Plain NiTi Textiles

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Abstract: Composite membranes with elastomeric matrix and reinforcement made of plain textile of NiTi wires can be considered as smart structures which inherited outstanding structural and functional properties of these wires. We present results of experimental studies based on inflation of such thin composite membranes – bulge test. The membranes are made of silicone rubber and their reinforcement are knitted fabrics or leno fabrics (perlinka). In these studies, three-dimensional digital image correlation (3D-DIC) technique is used for the determination of three-dimensional surface displacements and strain fields of the membrane surface. The method is proposed to determine also the main membrane curvatures and stress tensor fields in each surface point. Membrane stress tensor fields are determined from meridional and circumferential curvatures and the measured inflating pressure. Experimental results are compared with FEM simulation .

Keywords: Bulge test; 3D image correlation; Composite membrane; Reinforcement of SMA wires; Curvatures; Stress and strain analysis.

1. Introduction

Shape memory alloys (SMA) possess both sensing and actuating functions due to their shape memory effect, pseudo-elasticity, high damping capability and other remarkable properties. Because of these unique properties, SMAs are prospective in many applications such as structural vibration control, biomechanics and smart textiles. Combining the SMAs with other materials can create intelligent or smart composites by utilizing the unique properties of SMAs [1]. The experimental research and the modelling of smart elastomeric composites reinforced by SMA woven or knitted fabrics have been launched to determine the deformation behaviour of the material and then to identify material parameters of the appropriate constitutive equation.. Preliminary results of the experimental research of the elastomeric composites reinforced by different types of SMA knitted and woven fabrics are presented in this paper.

In the past years, we used the inflation of a thin cylindrical tube (cylindrical air-spring) in combination with the digital image correlation to examine mechanical properties of different types of rubber composites reinforced by textile cords. Similar material tests under non-uniform conditions were conducted [2] on plane circular composite membrane so called bulge test. The initially flat membranes were

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constrained between two circumferential clamps and subjected to a hydrostatic pressure. The digital image correlation system ISTRA was used. The system provides very rich experimental data recorded in files with Hierarchical Data Format (HDF5) which can be processed in common numerical computing environment like Matlab. Every session of ISTRA is recorded in the text file named series_master.issd in which the structure of each hdf5 file is described. The file contains data about undeformed and deformed three-dimensional surface geometry and about displacements and strains in discrete points on surface distributed in a regular grid.

Bulge test performed on the membrane of isotropic material provides an equibiaxial tensile stress state at the top of the inflated cap. It is supposed that a spherical shape exists in the neighborhood of summit and stress-strain state thus can be determined easily from measured inner pressure, membrane radius and stretch at the top. The inner force per unit length *T* can be determined from the equilibrium equation

$$\frac{2T}{R} = p, \tag{1}$$

where *p* is the inner pressure and *R* is the radius of the spherical surface.

However, the shape of the anisotropic bulge is rarely spherical. The basic structure of the fabric reinforcement has in general two main directions (warp and weft or course and wale) which results in orthotropic response and in the bulge cap shape resembling an ellipsoid. It is therefore possible to accept as a good approximation stresses for the membrane in the form of the ellipsoid [3]:

$$T_m = \frac{p}{2\kappa_c}, T_c = T_m\left(2 - \frac{\kappa_m}{\kappa_c}\right), \tag{2}$$

where T_m and T_c are inner forces per unit length and κ_m and κ_c are principal curvatures in meridional and circumferential direction respectively. The inner force components are expressed directly in terms of pressure and the current bulge geometry and they can be thus calculated directly from the experimental data.

2. Experiment

The knitted and woven fabric of SMA wires were made of commercially available drawn NiTi fibres Fort Wayne Metals Ltd. of the chemical composition 55.82 wt. % Ni giving the fibres superelastic behaviour at temperatures above 10^0C. The NiTi fibres of diameter 0.1 and 0.2 mm are produced by cold drawing. This technological process creates the fibres with a highly distorted microstructure with a high density of defects and with an occurrence of amorphous regions. The fibres with such microstructure do not show phenomena such as a shape memory effect and superelasticity which are typical for intermetallic alloys of nickel and titanium. To incite these properties it is necessary to recover microstructure of fibres by a heat treatment with appropriate time and temperature.

The fabrics were fabricated at the Dpt of Textile Structures of Technical University of Liberec. Knitted samples were made by hand on a flat knitting V-bed machine and leno weave fabrics samples were fabricated on weaving machine. Leno weave or doup weave (Fig.1) has two warp threads crossing over each other and interlacing with one filling threads which prevents shifting of fibres and minimises the distortion of threads.

The thermal processing of NiTi fabrics was performed at the Institute of Physics AS CR in Prague. Specimens of NiTi fabric 20x20 cm were treated for 30 min at 450°C in a resistance furnace at normal atmosphere stretched in two directions. The stress-strain graph of a fibre of diameter 0.1 mm submitted to the same thermal processing is shown in Fig. 2.

The composite specimens with silicon rubber matrix reinforced by NiTi fabric were prepared. NiTi fabrics were cleaned thoroughly in isopropyl alcohol. A dilute solution of triethoxysilane in the isopropyl alcohol has been used to increase an adhesion between the matrix and the NiTi fabric. The vacuum assisted resin transfer Molding with a rigid mould was used for the preparation of samples. The matrix of polyaditive silicone RTV ZA-13 was injected to the mould under 25 kPa vacuum. Due to this particular process, the resulting composite is devoid of bubbles and its quality is very good. The thickness of silicone membrane was about 3 mm.

Bulge tests were performed at room temperature. The samples of composite membrane were clamped in a test device between an aluminium vessel and an steel ring of inner diameter 144 mm (Fig. 3). A pattern consisting of chalk and coal dust was spread on the surface of membrane and fixed with hairspray. Pressurised air was introduced into the aluminium vessel, resulting to a bubble shape of the membrane. The pressure varying gradually by step 1 kPa between 0 and 90 kPa was recorded by a pressure sensor and the full-field bulge shape information was recorded by two digital cameras and processed by 3D - digital image correlation system Q400 of Dantec Dynamics. One pair of snapshots of deformed shape was captured in every step in the course of loading.

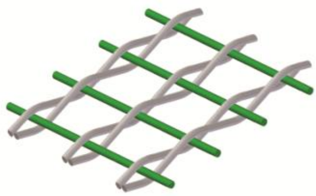

Fig. 1. Leno (doup) weave

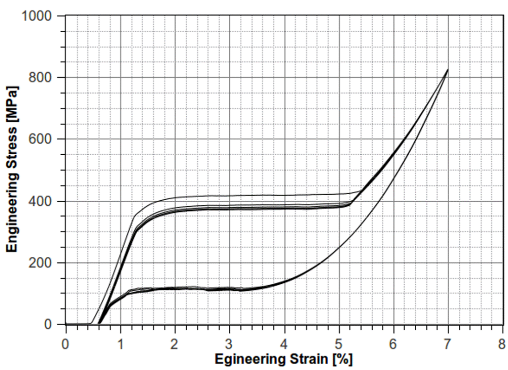

Fig. 2. Superelastic stress-strain response of NiTi fiber in tension

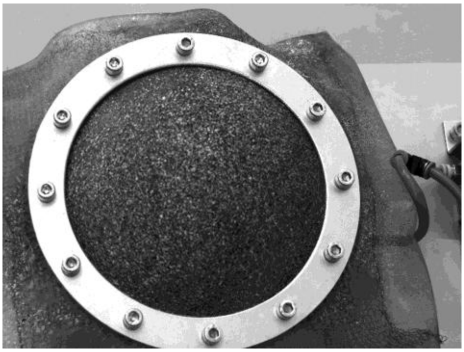

Fig. 3. Inflation of composite membrane

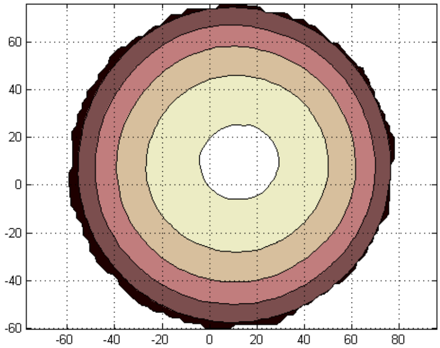

Fig. 4. Contour-lines at p=40 kPa of membrane reinforced by knitted fabric

Contour-lines (Fig. 4) of the deformed shape of the membrane reinforced by knitted fabric look at first sight as concentric circles centered at the pole. It could mean that the membrane has a spherical shape. The principal Green Lagrangian strains were calculated from the Istra data as mean values in grid points in the vicinity of the bulge pole. The graphs of the strains (Fig. 5 and Fig. 6) indicate that the shape of membrane is not spherical as assumed. The principal directions were found identical with the directions of course and wale of the knitted fabric or with the directions of warp and weft of leno weave fabric reinforcing the membrane. The ratio of membrane principal strains is approximately in 2:1 in the case of leno weave reinforcement which is the ratio of the main compliances of the leno fabric. The principal strain ratio in the membrane with knitted reinforcement is not as big since the values of stiffness in two main directions are not too different. The reconstruction of the deformed shape (Fig. 7) of the membrane reinforced by knitted fabric (Fig.8) shows several outstanding or missing points where the correspondence between the stereo pairs acquired before and after deformation was not determined.

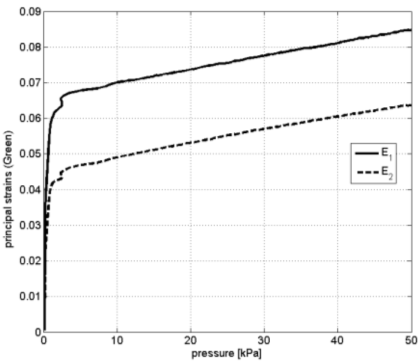

Fig. 5. Principal strains - knitted reinforcement

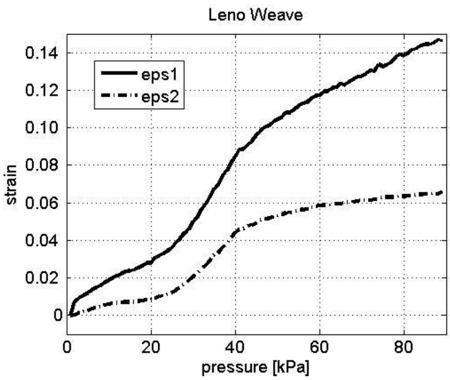

Fig. 6. Principal strains - leno weave reinforcement

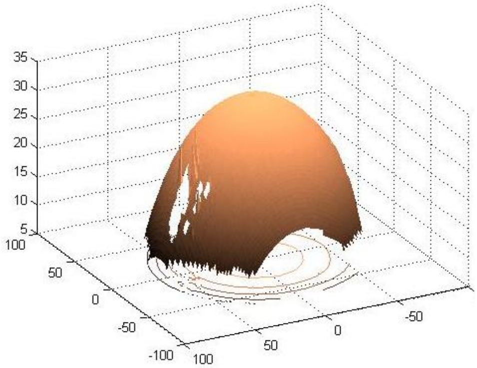

Fig. 7. Shape of deformed membrane

Fig. 8. Membrane of silicon rubber reinforced by knitted fabric made of NiTi wires

3. Experimental data processing

System ISTRA Q400 provides Cartesian coordinates x, y, z of a set of physical points M on the deformed membrane surface. The global reference system is defined during the stereo rig calibration phase. The calibration is done by taking a series of exposures of a calibration target which is a chess pattern with known geometry. The position of calibration target which was recorded as the first determines the centre and the orientation of the coordinate system. Axes of this coordinate system do not lie in general in the principal planes of the membrane cap.

It is possible to approximate an analytic form of the surface within some defined neighbourhood of the top of the inflated cap and then to estimate the local coordinate system and the curvatures at this point. We tested two possible manner to express this approximation - as an implicit function $F(x,y,z) = 0$, or as a parametric representation $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$. The local analytic approximation of the membrane can be chosen as a quadric surface defined by the equation:

$$F(x,y,z) = a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0 \quad (3)$$

or in vector and matrix notation:

$$x^T A\, x = 0, \quad x^T = \{x, y, z, 1\}, \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ & a_{22} & a_{23} & a_{24} \\ & & a_{33} & a_{34} \\ sym & & & a_{44} \end{bmatrix}. \quad (4)$$

The coordinates (x_i, y_i, z_i), $i = 1,\dots,N$ will be substituted into Eq.3 and the ten coefficients $a_{11},...,a_{44}$ will be determined from the system of *N* linear homogeneous equations. Denote *N*x*10* matrix *M* as:

$$M = \begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & x_1 y_1 & x_1 z_1 & y_1 z_1 & x_1 & y_1 & z_1 & 1 \\ . & . & . & . & . & . & . & . & . & . \\ x_N^2 & y_N^2 & z_N^2 & x_N y_N & x_N z_N & y_N z_N & x_N & y_N & z_N & 1 \end{bmatrix},$$

then the coefficients $a_{11},...,a_{44}$ will be equal to the components of eigenvector corresponding to the smallest eigenvalue of the matrix $Q=M^TM$. The coefficients will be substituted into the matrix *A* (Eq. 4) and the axes of the local coordinate system will be determined as eigenvectors of the submatrix *B* of matrix A :

$$B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ & a_{22} & a_{23} \\ sym & & a_{33} \end{bmatrix}. \quad (5)$$

The principal curvatures κ_1 and κ_2 of the surface $F(x, y, z) = 0$ defined implicitly can be calculated [4] from the Gaussian curvature K_G and the mean curvature K_M which depend on the gradient ∇F , the hessian *H(F)*, and the adjoint of the hessian $H^*(F)$:

$$\nabla F = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \end{pmatrix} = \begin{pmatrix} F_x & F_y & F_z \end{pmatrix},$$

$$H(F) = \begin{pmatrix} F_{xx} & F_{xy} & F_{xz} \\ F_{yx} & F_{yy} & F_{yz} \\ F_{zx} & F_{zy} & F_{zz} \end{pmatrix} = \nabla(\nabla F), \quad (6)$$

$$H^*(F) = \begin{pmatrix} Cof\ F_{xx} & Cof\ F_{xy} & Cof\ F_{xz} \\ Cof\ F_{yx} & Cof\ F_{yy} & Cof\ F_{yz} \\ Cof\ F_{zx} & Cof\ F_{zy} & Cof\ F_{zz} \end{pmatrix},$$

where $Cof\ F_{xx}$ is the cofactor of the element F_{xx} of the hessian matrix *H(F)*. Therefore, the unit normal *N(F)*, curvatures K_G and K_M and principal curvatures κ_1 and κ_2 at each point of the surface can be calculated as follows:

$$N(F) = \frac{\nabla F}{|\nabla F|}, \quad (7)$$

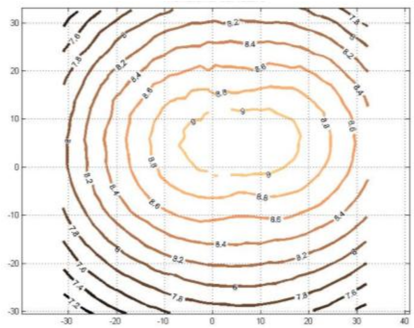

Fig.9. Contour-lines of meridional curvature $[m^{-1}]$, p=60 kPa, leno weave reinforc.

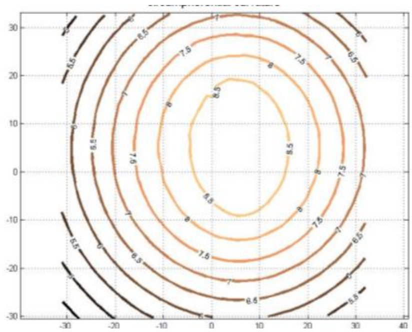

Fig. 10. Contour-lines of circumferential curvature $[m^{-1}]$, p=60 kPa, leno weave

$$K_G = \frac{\nabla F \; H^*(F) \, \nabla F^T}{|\nabla F|^4} = \frac{\begin{vmatrix} H(F) & \nabla F^T \\ \nabla F & 0 \end{vmatrix}}{|\nabla F|^4},$$

$$K_M = \frac{\nabla F \; H(F) \, \nabla F^T - |\nabla F|^2 \, Trace(H)}{2|\nabla F|^3}, \qquad (8)$$

$$\kappa_1, \kappa_2 = K_M \pm \sqrt{K_M{}^2 + K_G}.$$

All these calculi in matrix form were implemented in Matlab environment and the experimental data were processed almost automatically. The principal curvatures at the top of the deformed shape of membrane reinforced by leno weave NiTi fabric (Fig. 9 and 10) show only small differences in the numeric values. However, the shape of their contour lines differs significantly of the circular one because

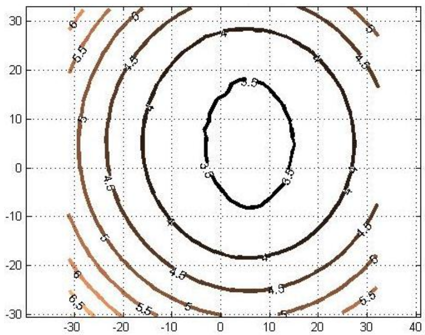

Fig. 11. Contour-lines of meridional force [N/mm], p=60 kPa, leno weave reinf.

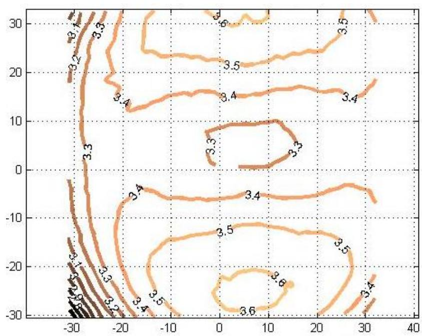

Fig. 12. Contour-lines of circumferential force [N/mm], p=60 kPa, leno weave reinf.

of different compliances in the two main directions of the reinforcement. The inner forces per unit length in circumferential and in meridional direction (Fig. 11 and 12) were determined according the formulas of Eq. 2. The results show again slightly non-uniform distribution of forces per unit length in composite membrane.

4. Conclusion

The experimental data containing the geometry of deformed shape of composite membrane, acquired by the 3D digital image correlation method, were processed in Matlab and the principal curvature and the inner forces were calculated. This process involved some basic instruments of differential geometry which are included in this paper. The formulas for geometry features mentioned are suitable for matrix operations in most of mathematical software. Further useful information and method can be found in references [4, 5]. Machado's paper [6] includes a functioning Matlab subroutine for calculation of principal curvatures of surface in parametric form which requires surface coordinates in the form of grid with constant step.

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