

Fracture Process Analysis in Silicate-Based Composite Specimens

Petr Frantík¹, Václav Veselý² & Zbyněk Keršner³

Abstract: Estimation of the size and shape of the zone of material failure developing around the crack tip during fracture of silicate-based composites presents the main focus of the paper. Numerical simulations utilizing two different modelling approaches (lattice-particle type model and finite element model) are used for verification of a semi-analytical technique for the inelastic zone determination developing by the authors. The problem is studied via virtual three-point bending tests on notched beams of two considerably different sizes made of silicate-based materials with three different compositions resulting in significantly dissimilar cohesive behaviour (from quasi-brittle to quasi-ductile/strain hardening). Good agreement between results of the semi-analytical technique for the estimation of the fracture process zone extent and the simulations with discrete model is reported.

Keywords: Fracture; Silicate composite; Process zone, Lattice-particle model

1. Introduction

The paper is focused on estimation of the size, shape and other relevant properties of the zone of material fracture evolving at the tip of propagating crack in quasi-brittle building materials, particularly cementitious composites. Characteristics of the zone, in the case of the materials in question referred to as the fracture process zone (FPZ), are prospected to be utilized within methods for evaluation of fracture mechanical parameters in order to decrease the effects of the test specimen's size, geometry and free boundaries on their determined values. A determination procedure accounting for the mentioned effects is under development by the authors and outputs of individual parts of the procedure are tested and validated. This paper presents some results on numerical verification of the part of the technique which is responsible for the estimation of the size and shape of the FPZ. Numerical simulations of the fracture process in the test specimens via a lattice-particle (spring network) model defined as dynamical system is used for the verification. Materials with considerably different cohesive behaviour (classical quasi-brittle softening, with high strain capacity, and even strain hardening) are considered in the research. Experimental validation of the FPZ is planned with help of either acoustic emission (AE) technique or radiography. Results of the presented numerical simulations are particularly suitable to be validated using AE scanning as the manner of the material fracture simulation

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using the discrete-type model is considered to be similar to the cracking events triggering AE.

2. Motivation

The research is motivated by a need for an appropriate characterization of the above-mentioned materials from the perspective of the tensile failure by means of a unique set of parameters which do not depend on the size and shape of the test specimen(s) and laboratory set-up/conditions, see e.g. [1] as a representative reference. Such parameters shall serve as relevant inputs into computational tools for simulations of the structural behaviour and characterize the material's ability to resist the failure propagation.

Recently, the authors have been developing such a technique (termed the ReFraPro – Reconstruction of Fracture Process) which shall in particular be applied on the determination of the fracture properties of cementitious quasi-brittle materials from experimental tests on laboratory specimens [2]. The possibilities of specification of the energy dissipated within the FPZ by the volume of the FPZ rather than by the cracked ligament area, as is usual in the RILEM work-of-fracture method [3] have been investigated and proposed for the developing procedure [4]. An amalgamation of classical nonlinear fracture models for concrete, multi-parameter fracture mechanics and plasticity theory is employed for a reconstruction of the FPZ volume by which the amount of the energy consumed during the fracture process should be specified.

The technique has been already partially validated [2,5] using published results of experimental measurements – AE scanning (and also indentation tests [6]). However, only a limited number of works on this topic is available, typically with slightly different focus or containing data that are not directly usable for its sound validation. Moreover, they are often concerning only with normal concrete or mortar, i.e. quasi-brittle material with characteristic length in the order of magnitude of units of cm/dm, not fibre- or otherwise reinforced, i.e. quasi-brittle/quasi-ductile material of characteristic length of units of meters. Therefore, numerical simulations of fracture tests have been carried out to gain better evidence and understanding/verifying of fundamental issues for the developing ReFraPro method.

Simulation methods are also of great importance in connection with the mentioned experimental analyses, namely the fracture monitoring via AE measurements. In this field, the results of numerical simulations can reveal lot of aspects of the real failure mechanisms taking place in the material of the loaded specimen. For that reason detailed numerical analyses of failure propagation accompanied by AE have been also conducted by the authors [7,8]. This paper relates to those phenomena as well. Computational tools based on physical discretization of continuum (a model from the class of lattice-particle models) and finite element method with implementation of a cohesive crack model have been employed. The results of such numerical simulations are considered to answer questions regarding the evolution of the FPZ, the amount of energy dissipated within it, etc.

3. Modelling methods

3.1. Estimation of fracture process zone extent – semi-analytical technique

The part of the ReFraPro procedure – the FPZ extent estimation – is based on three fundamental approaches to model material failure. It exploits the theory of linear elastic fracture mechanics (LEFM), classical nonlinear fracture models for concrete, i.e. the effective crack model (ECM, [9]) and the fictitious crack model (FCM, [10]), and plasticity theory. For the general description of the stress field in cracked bodies, more terms of the Williams power series [11] are taken into account. It is, therefore, referred to as multi-parameter LEFM in this context. A detailed description of the method of FPZ extent estimation is given in [2]; here we only sketch out the fundamental ideas behind the procedure.

The method is utilized within the processing of fracture test records; typically, load–displacement (P – d) diagrams. For individual stages of the fracture process, the length of the equivalent elastic crack is estimated by means of the ECM. Then, the stress state in the body with the effective crack is approximated through the Williams power series. Subsequently, the extent of the zone where the until-now elastic material starts to fail is determined by comparing the tensile strength f_t of the material to a relevant characteristic of the stress state around the crack tip (some sort of equivalent stress, σ_{eq} , for cementitious composites e.g. the Rankine or Drucker-Prager failure criterion can be employed). This zone referred to as Ω_{PZ} (index PZ stands for Plastic Zone). Then, in agreement with the cohesive crack approach, the FPZ (marked as Ω_{FPZ}) is supposed to extend from the Ω_{PZ} around the current crack tip up to an Ω_{PZ} corresponding to the prior crack tip (i.e. currently a point at the crack faces), where the value of crack opening displacement w reaches its critical value w_c (i.e. the value of cohesive stress $\sigma(w)$ drops to zero here). Thus, the region Ω_{FPZ} is assembled as a union of Ω_{PZ} within the field of activity of the cohesive stress. In order to project the shape of the cohesive stress function $\sigma(w)$ into the shape of the Ω_{FPZ} , the individual plastic zones $\Omega_{PZ,i}$ creating the entire FPZ are scaled by a factor of $\sigma(w_i)/f_t$ in this method. The envelope of the Ω_{FPZ} zones for each stage of the fracture represents a region in which some sort of damage has taken place during the fracture process throughout the entire specimen ligament, referred to as the cumulative damage zone (CDZ), Ω_{CDZ} .

3.2. Verification via physical discretization of continuum modelling and cohesive crack modelling – numerical simulations

Physical discretization is based on the idea that a phenomenon can be studied on an alternative structure which can be much more easily modelled as a numerical representation by the computer than the original structure. This procedure is well known in the theory of nonlinear dynamical systems, where it leads to extremely simple models. In this case, the modelled specimen is substituted by a set of mass points mutually connected by simple translational springs. They load the neighbouring mass points by forces whose value depends on the elongation of the springs. The springs' behaviour can be generally defined as fully nonlinear; in this case a multi-linear force–elongation function was used. The model is then defined as a nonlinear

dynamical system described by equations of motion including damping. The technique for the continuum discretization within this model is based on a procedure developed for the rigid body spring networks [12]. This means that the mass of the mass points and the parameters of the translational springs are computed based on Delaunay triangulation and Voronoi tessellation. Every mass point has its own Voronoi cell, which specifies its mass according to cell area, material density and thickness. Similarly, every translational spring has an edge of a Voronoi cell which specifies its ‘cross-sectional’ area. This area is then used for calculation of the spring parameters. More details about the used computational and discretization procedures (program FyDiK [13]) can be found in [5].

For the numerical simulations of the quasi-brittle fractures also the commercial Atena finite element software [14] was utilized. This tool is based on principles of nonlinear fracture mechanics, damage and plasticity and in both 2D and 3D versions is widely used in engineering computations. The authors already performed numerical analyses of fracture propagation in several testing configurations with help of these modelling approaches, even for the case of AE events simulation [7,8].

4. Virtual experiment

Issues related to the damaged material extent are investigated on virtual tests on notched beams of two sizes subjected to three-point bending (TPB). Three material sets differing in the cohesive behaviour were included in the study, which is a reason why two noticeably different sizes of specimens were considered – the shape of cohesive law influences the structural response of small and large beams in different way.

4.1. Geometrical models, discretization techniques

The scheme of the test geometry is shown in Fig. 1; the dimensions of the considered beams are summarized in Tab. 1. The mid-span relevant parts of the geometrical models corresponding to the used computational tools are depicted in Fig. 2. The mesh density for both the small and large beam within the either simulation technique was used constant: the element size of 8 mm in the case of Atena FE models, the average mass points distance of cca 5 mm in the case of FyDiK models.

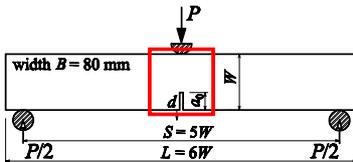


Fig. 1. Test geometry

Table 1. Specimen dimensions

in [mm]	W	B	S	L	a_0	$\alpha_0 = a_0/W$
SIZE I	160	80	800	960	16	0.1
SIZE II	640	80	3200	3840	64	0.1

4.2. Material models

Three materials differing only in cohesive behaviour, see Fig. 3 left, were considered: *i*) a common quasi-brittle material with bilinear cohesive function, *ii*) a material of the sort of fibre reinforced composites with trilinear cohesive function, and *iii*) a strain hardening fibre-reinforced material with trilinear cohesive function.

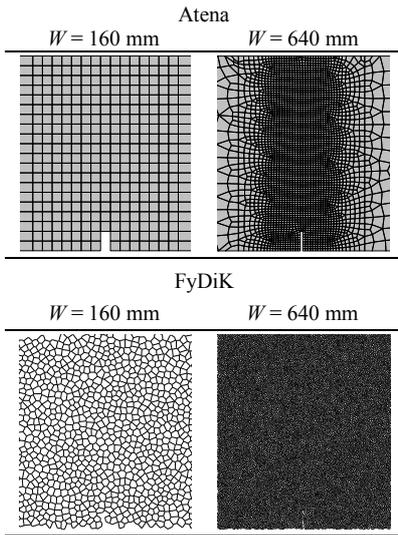


Fig. 2. Meshes of mid-span parts of the Atena FE model and FyDiK lattice-particle model, for the small and large TPB specimen, resp.

model was created from a homogeneous elastic isotropic material with parameters corresponding to the reference material set (used for Atena simulations).

Their other mechanical/physical properties are considered in a way corresponding to the (default) material definition for the Atena software which uses recommended values/functions from design codes and other relevant documents [14].

Atena – In this software, a set of mechanical, fracture and physical parameters was generated for a concrete of cubic compressive strength $f_{\text{cub}} = 85$ MPa. This material set was then held as the reference set. The basic parameters of this set are as follows: specific gravity $\rho = 2400$ kg/m³, modulus of elasticity $E = 44.5$ GPa, Poisson’s ratio $\nu = 0.2$, compressive strength $f_c = 85.0$ MPa, tensile strength $f_t = 5.17$ MPa. The value of fracture energy depends on the shape of the cohesive function.

FyDiK – A simple multi-linear force–elongation function (with multi-linear descending branch in tension, according to Fig. 3) has been utilized. The numerical

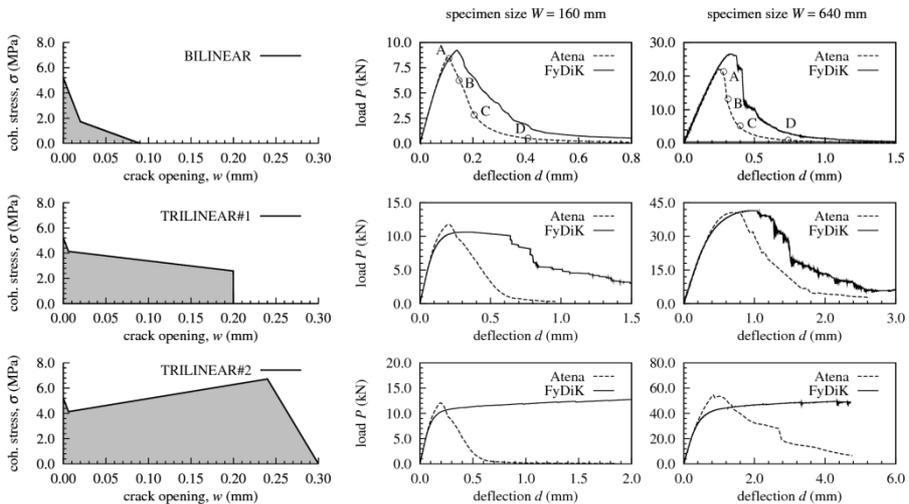


Fig. 3. Cohesive functions considered in the simulations (left); comparisons of the P – d diagrams simulated by different used models for the corresponding cohesive functions and specimen sizes, respectively (middle and right)

5. Discussion of results

The P - d diagrams from both the FyDiK and Atena simulations are plotted in Fig. 3 in the middle and right column for the small and large specimen, respectively. Reasonable agreement between the curves corresponding to the two modelling approaches is observed only for the bilinear cohesive function.

The P - d diagrams from the Atena simulations are regarded as records of real experimental test in this study. As such, they are used as inputs into the ReFraPro technique for the estimation of the FPZ and CDZ extents. Other inputs into the procedure are following: 4 terms of the Williams series, Rankine failure criterion, and the corresponding cohesive function. The FPZ extents (including the indication of the cohesive stress distribution, see the scale in MPa) in 4 stages of the fracture process (marked by letters A, B, C and D in the P - d curves in the top row in Fig. 3) in the specimens made of material with bilinear cohesive function are shown in Fig. 4. This figure contains also the FyDiK interpretations of the FPZ, i.e. the union of the failure events within the cohesive function impact. The failure events with low energy release (less than 2 mJ) are filtered.

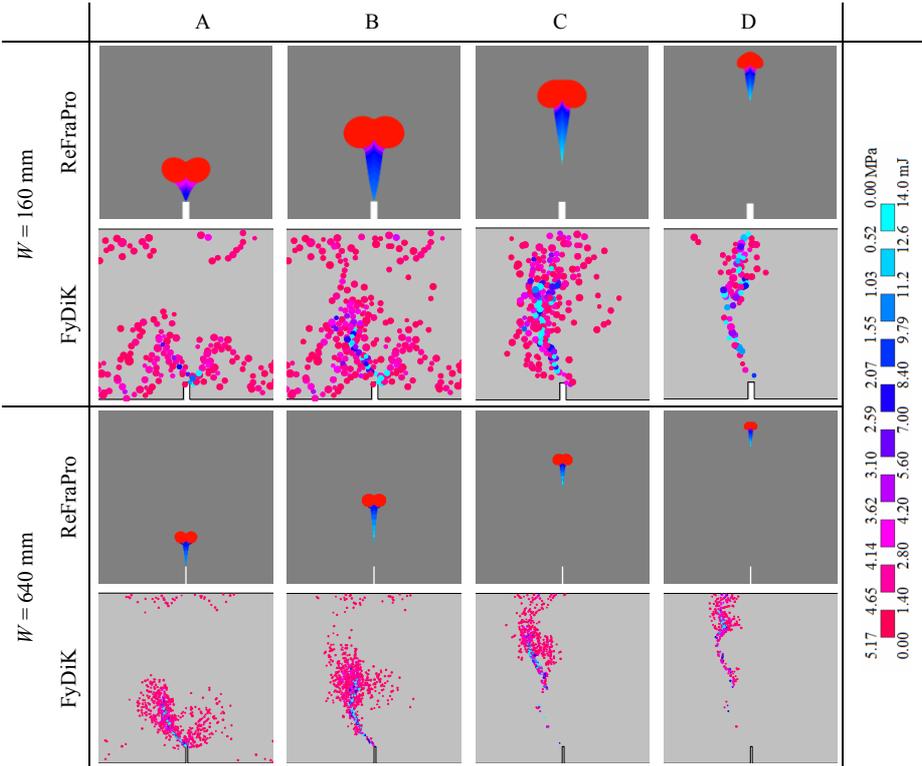


Fig. 4. FPZ extents estimated by the ReFraPro method and the FyDiK model for the bilinear cohesive function for 4 stages of the fracture process and two specimen sizes (the middle part of the beam is displayed); the cohesive stress intensity and the energy dissipation intensity, respectively, is indicated (see the scale on the right)

The extents of the CDZ for both small and large beams and all considered cohesive functions estimated by ReFraPro technique are depicted in Fig. 5. They contain also the results of FyDiK simulations – the representations of the CDZ, i.e. the cumulative number of failure events (including the indication of the intensity of the energy dissipation, see the scale in mJ). The CDZ estimated by the ReFraPro technique and FyDiK simulations can be compared here also to the interpretations of the same phenomena obtained using the Atena software (displayed as areas with cracks).

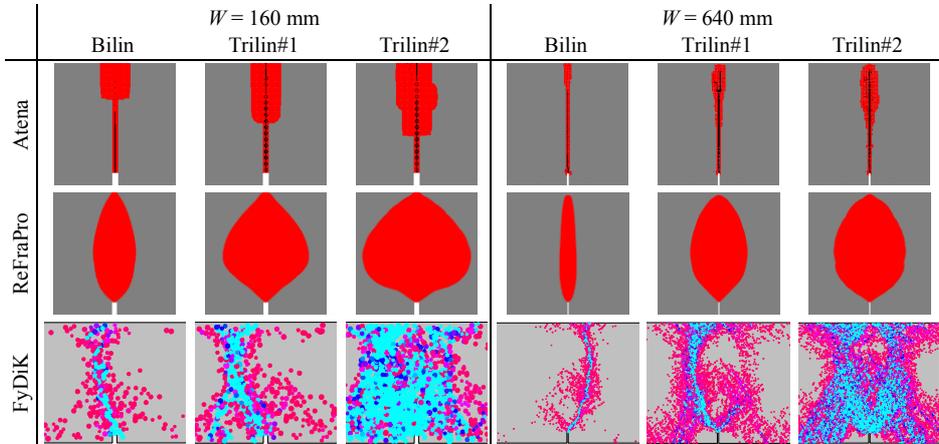


Fig. 5. CDZ extents estimated by the Atena simulation, the ReFraPro method, and the FyDiK model for three cohesive functions and specimen width 160 mm and 640 mm, respectively

6. Conclusions

The paper presents an analysis aimed at the verification of the (semi-) analytical technique for estimation of the extent (size and shape) of the FPZ and the CDZ in quasi-brittle silicate-based composites during tensile failure. The method was applied on virtually tested materials exhibiting also rather ductile, even strain hardening, cohesive behaviour. The FPZ extents estimated using the ReFraPro and FyDiK approach are in reasonable agreement, both in the characteristics of the size and shape and the parameter describing the failure (the intensity of cohesive forces and energy dissipation, respectively). The extent of the overall failure zone estimated using the ReFraPro technique agrees with the FE and lattice-particle model simulations quite well. The area of the damaged material resulting from the Atena simulation is considerably narrower and of different shape than the ones estimated by the ReFraPro method and the FyDiK simulations.

The used FyDiK model provided a fairly good approximation of both the current and cumulative failure volume, including their energy consumption. Results of such simulations can serve as a significant source of new knowledge which can be obtained regarding the energy dissipation mechanisms in the FPZ in the case of quasi-brittle materials. Nevertheless, it should be noted that the conclusions coming

out from the comparisons should be regarded as provisional. Subsequent analyses of the results are planned. This research shall include studies regarding the cohesive stress distribution over the FPZ, with which the energy dissipation is connected. The analysis using the FyDiK model can be considered as a virtual AE scanning result.

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