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On the Coherence of Mathematical and Experimental Methods in Structural Analysis

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Abstract: New achievements in physics and electronics, inventions in Laser- and LED-techniques, new recording systems like CCD-cameras, but above all the availability of computer-techniques, both in methodology and equipment enhanced the attraction of experimental mechanics, its position and importance in structural engineering beside mathematical/numerical methods. However the performance of experimental structural analysis as well as of the mathematical/numerical modeling and analysis is full of error-sources, impairing accuracy and reliability of results. To overcome the disadvantages both these processes should be combined therefore, considering the coherence between them. This requires the fully exploitation of the measurements and immediately an additional mathematical problem is set, the solution of inverse problems.

Keywords: Reliability of structural analysis, mathematical / experimental coherence; inverse problems.

1. Introduction

Pursuing the developments in experimental mechanics over the past century, remarkable progress and essential changes in measurement methods are to be noted, based on new achievements in physics. Laser- and LED-techniques had been invented and brought up new optical methods like holography in its different concepts, speckle-interferometry, electronic-speckle-interferometry and digital correlation technique in manifold variations. Remarkable progress is to be noted in improving photo-elasticity- and Moirè-techniques in their various concepts. High-tech developments in the instrumentation of electrical measurement methods have made their application easier. New sensor-techniques have widened the catalogue of measurement-techniques, explicitly to mention the fibre-optical techniques. And concerning the optical methods the introduction of CCD-cameras had revolutionised experimental mechanics.

But the most effective influence on the developments in measurement techniques and on the experimental mechanics at all is to put on the introduction of computer-techniques and the appropriate software in a broad variety. Konrad Zuse had constructed in 1941 the first programmable computer, followed by unbelievable rapid further developments in computer-techniques world-wide. And soon engineers appreciated the possibilities and potentials; they immediately used them to solve their mechanical problems, rendering them into finite formulations and modelling

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the problems by means of numerical, mainly FE-methods. And it looked as if with the possibility of FE-modelling of engineering problems and related computer-programs any technical task, - and not technical ones only -, could be solved. In this concern the question had been raised, whether one could refrain from experimental mechanics and analysis. But in the contrary it must be stressed that measurement and experimental mechanics has achieved extended importance just because of the advances in methodologies and equipment related to computer-techniques and their increasing application in overcoming the challenges in mechanical engineering.

2. Assessment of mathematical/numerical analysis.

Mathematical/numerical analysis of technical problems based on FE-modelling is an idealisation and a simulation process, including many uncertainties like choice of element-types, the structure and fineness of mesh, missing information on parameters, variability of variables, inaccurate knowledge on non-linear and time-depending responses, modelling of boundary conditions etc. Beside three major causes responsible for biasing the results of numerical analysis are to take into consideration:

i) Missing verification and validation of the mathematical model. According to *NATKE* [1] "*verification is to define as the reconstruction of the information used for the construction of the model, which can concern the principles of mechanics and/or the measured data used, whereas validation is to define as the proving of the homomorphy between system and model, which includes the structure in addition to the input/output quantities*."

ii) Numerical uncertainty in the classical computer-arithmetic because of the error propagation in the solution, based on billions of arithmetic operations, see e.g. floating point operations.

iii) Unreflecting use of computer-programs, especially commercially available ones, without knowing their mathematical background and range of validity, withdrawn from any control of the codes and with that believe in the infallibility of computer calculations and in the correctness of results.

3. Assessment of experimental analysis

On the other hand experimental analysis and testing provides a means of validating the mathematical/numerical model and immediately information on the object/structure responses like deformations, displacements, strains, frequencies, modes etc. probably at least hints on non-linear, time-depending and other effects. This is advantageous, as some effects are difficult to simulate accurately if at all in mathematical modelling.

But like in mathematical/numerical analysis also in experimental analysis inaccuracy and uncertainties are to take into consideration. In principle all measured data are full of systematic and random errors. These may be caused e.g. by improper calibration of instruments in the experimental set-up and its mounting, neglecting the specific measuring range of the single instruments in the system, their signal-to-noise ratio, the impedances along the path of signal transmission. Furthermore environmental influences on object and measurement system as well as improper inclusion of real boundary- and loading-conditions may be responsible for erroneous data.

In experimental mechanics computer-techniques as well play a decisive role in connection with the improvements in the well-known methods and especially with regard to interferometric methods. The measurements yield optical signals in kind of interferograms, which are recorded by high-dissolving CCD-cameras. These interferograms have to pass through different mathematical processes like Fourier- and wavelet-transformation, phase-shifting, phase-unwrapping, filtering etc. before finally the deformations sought after are obtained. Because of the tremendous amount of data recorded in the interferograms the performance of such processes requires proper image-processing software. And consequently some critical remarks referring to mathematical analysis apply to computer-aided evaluation of interferograms. It must be pointed out that in addition the basic concept and the theoretical background of hardware and related software as well as the limits of applicability of the used measurement system must be known and understood.

4. Combination of mathematical and experimental processes

To overcome the disadvantages as mentioned above, i.e. to reduce the sources of errors, to verify and to validate the mathematical model and to improve the results of structural analysis the best means as mentioned already by *PRYPUTNIEWICZ* [2] is to combine mathematical and experimental concepts to a comprehensive procedure as shown in Fig.1.

That is turning away from the as yet inductive and deductive procedures to a unite concept of experiment and simulation, thus meeting in consequence the postulate "*to pay absolutely attention to the principle "praxis cum theoria" for all systems in the open nature*" (*ANGER*, [3]). From the mechanical point of view the combined procedure can be considered as the application of system identification. This immediately leads to improperly-posed problems, the solutions of which require additional mathematically ambitious processes.

The experimental analysis including the inverse solutions yield information on parameters, variables, assumptions and further more on stress states, however depending on the initial mathematical model. These results are to compare with the results of the mathematical analysis to ascertain the degree of matching. Corresponding to this degree the parameters, variables, assumptions, constraints, boundary- and loading-conditions are updated, the FE-mesh modified and inconsistencies eliminated. Thus the mathematical model and with that the operator-matrix will be improved till the final results are in acceptable correspondence. Then the model can be considered as verified and validated unless the applied FE-program itself has proved its reliability. But nevertheless it remains necessary that the analysts must have a critical look at all of the applied computer-programs; they should know at least the backgrounds and the underlying mathematical coherences.

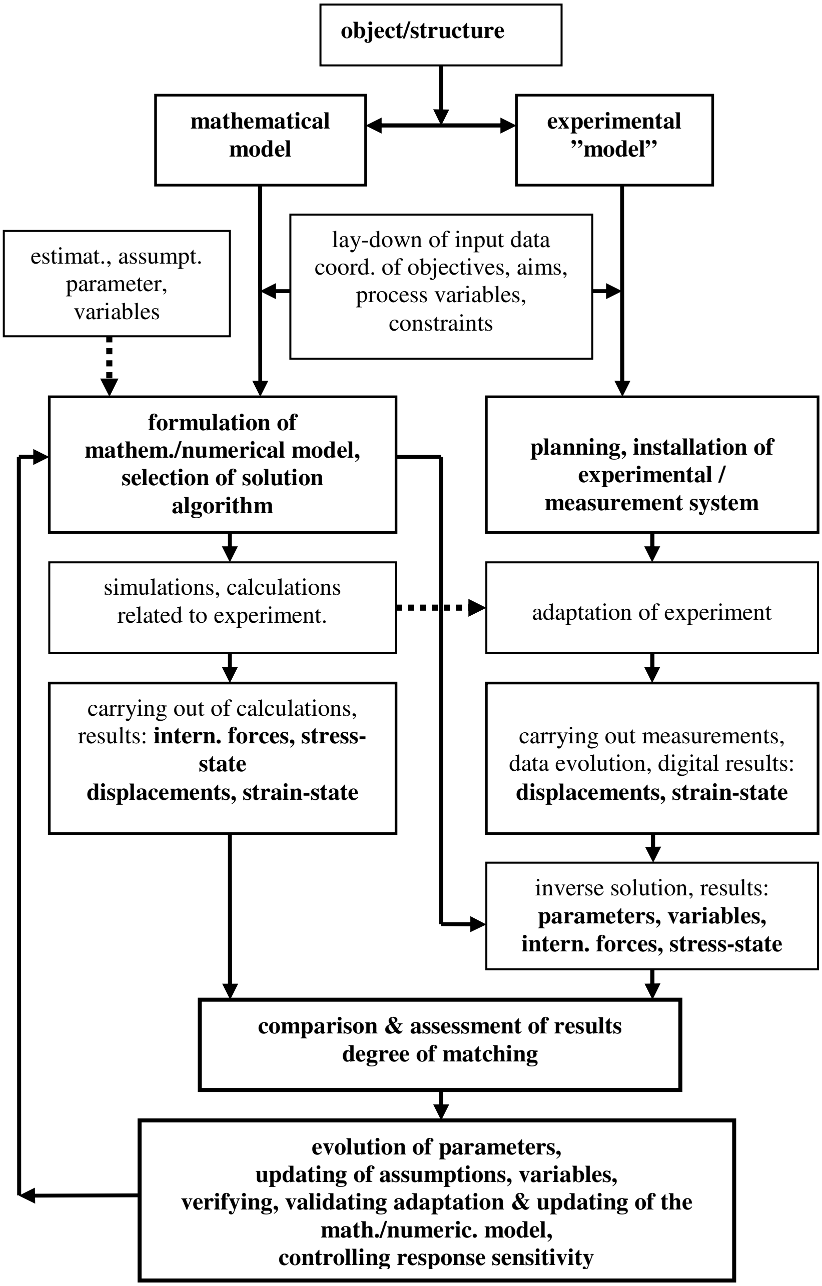

Fig.1. Concept of combining mathematical and experimental processes in structural analysis.

5. Definition of inverse problems

The measurements first of all yield analogue “signals” like electrical, optical, acoustical, radiation- and radio signals. However these signals and the data of deformations derived out of them do not come up to information, which are necessary for relevant analysis and assessment of the problem considered, because the parameters necessary for comprehensive system identification and model validation cannot be measured directly. They are to calculate on the basis of the experimental data and the operator matrix corresponding to the computational model of the object.

Any structural mechanical problem can be described by the relation

$$\mathbf{y} = \mathbf{A}(\mathbf{p}) \cdot \mathbf{x} \qquad (1).$$

In experimental mechanics let **y** denote the M-dimensional vector of effects, the elements resulting from measurements. The N-dimensional vector **x** denotes the vector of causes; the elements include for instance loading, boundary conditions, constraints for example. The ($M \times N$)-dimensional operator-matrix $\mathbf{A}(\mathbf{p})$ represents the mathematical and computational model of the object/structure and relates the input signals to the output signals. Depending on the information sought either a forward or an inverse problem is to solve (Fig.2).

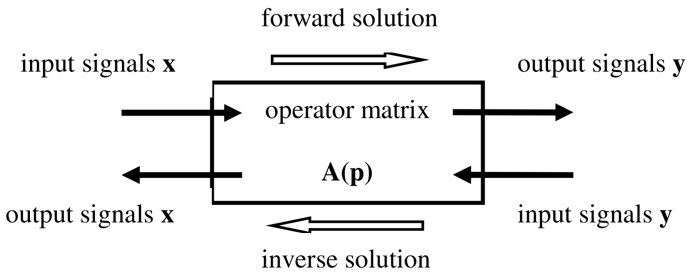

Fig. 2. Forward/inverse solution

Provided **A** and **x** to be given, the Eq. (1) describes a direct problem, which always leads to a well defined solution, no matter, whether the mathematical model depicts the reality or not, provided a forward-solver is on hand. However concerning the above described problem of updating and adaptation in order to verify and validate the mathematical/computational model, the elements of the operator-matrix **A** are unknown because of unknown or at least partly unknown parameters and moreover **x** and **y** are incomplete, the letter because of missing data points, unreliable and lost data etc., a mixed inverse problem is on hand [4].

6. Inverse solution algorithms.

All inverse problems can be seen as fitting a hypothesised model to measured data in order to estimate unmeasured quantities or immeasurable ones respectively. Generally they are improperly posed, because the operator-matrix is not a regular, positive defined square matrix and thus cannot be inverted. Yet a pseudoinverse can be found always. The solution of inverse problems is mathematically ambitious and adds a new chapter of mathematics to comprehensive structural analysis [5-7]. Numerous methods are known, Some of them, which have been proved to be quite useful in practical application [8], are listed up in Fig.3.

Matrix inversion methods

Least-Squares-Solution, Minimum-Length-Solution, Tikhonov-Regularisation, Damped-Least-Squares-Solution, Tikhonov-Miller-Regularisation, Truncated Singular-Value-Decomposition;

Iterative methods

Algebraic-Reconstruction Technique, Sequential-Image-Reconstruction Technique, Landweber-iteration; Modified Matrix-Inversion-Solution

Simulation methods

Artificial Neural Networks (e.g. Multi–Layer-Perceptron), successive forward simulation, Monte-Carlo approach, genetic algorithms, fuzzy-logic

Fig.3. Methods for solution of inverse problems.

As a matter of fact solutions of inverse problems are not unequivocal from the first, in principle leading to "families" of solutions. Different methods applied to the same problem can lead to completely different answers. But although the calculated results solve the initial equations, i.e. Eq. (1), they do not render necessarily the considered problem. Therefore it is of outmost importance to proof always, whether the selected solution is physically meaningful within the context of the engineering problem. Incorporating additional à-priori information on the subject considered, furnished by the experience of the analyst, is recommended. However it must be checked carefully that they do not bias the results and lead to incorrect conclusions. With reference to those solution methods, which demand initial estimates of the quantities sought after, the estimates should be as close as possible near the reality.

7. Principle of the Sensitivity-matrix-based method

For solution of mixed inverse problems it is recommendable to apply the *Sensitivity-matrix-based method* [8, 9]. She enables an effective updating and adaptation of the computational model including parameters, variables and assumptions etc. The method can be categorised as an iterative method. For estimated N-dimensional vectors $\mathbf{x}^{(\mu-1)}$, $\mathbf{x}^{(\mu)} = \mathbf{x}^{(\mu-1)} + \Delta\mathbf{x}$ the forward solver Eq. (1) yields the M-dimensional vectors $\mathbf{y}^{(\mu-1)}, \mathbf{y}^{(\mu)} + \Delta\mathbf{y}$, the relation between these vectors runs

$$\mathbf{y}^{(\mu)} = \mathbf{y}^{(\mu-1)} + \mathbf{S}\left(\mathbf{x}^{(\mu)} - \mathbf{x}^{(\mu-1)}\right) \tag{2}$$

The sensitivity matrix $\mathbf{S}$ relates finite changes of the vector $\mathbf{x}$ of unknown quantities to finite changes of vector $\mathbf{y}$.

$$\mathbf{S} = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \tag{3}$$

The measured values $\mathbf{y}^{(meas)}$ are different from the estimated $\mathbf{y}^{(\mu)}$. Minimising the norm of the residuals leads to the relation

$$\Delta\mathbf{y} = \mathbf{S} \cdot \Delta\mathbf{x} \tag{4}$$

The sensitivity matrix is not a regular, positive defined square matrix and thus an inverse does not exist. Yet a pseudo-inverse can always be found by means of the methods described above and with that Eq. (4) runs

$$\Delta\mathbf{x} = \mathbf{S}^{-g} \cdot \Delta\mathbf{y} \tag{4a}$$

Let in case of a mixed inverse problem the vector of causes **x** be unknown as well as the vector **p**, included in the operator-matrix, then Eq. (4) holds

$$\Delta\mathbf{y} = \begin{bmatrix} \mathbf{S}_{(x)} & \mathbf{S}_{(p)} \end{bmatrix} \times \begin{Bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{p} \end{Bmatrix} \tag{5}$$

To solve Eq. (5) the iterative process starts with proper estimates $\mathbf{x}^{(0)}, \mathbf{p}^{(0)}$. Step by step the increments $\Delta\mathbf{x}$, $\Delta\mathbf{p}$ are calculated and **x** and **p** are updated unless the calculated values of **y** coincide with the measured data $\mathbf{y}^{(meas)}$. This turns out to be an essential part of updating and adapting parameters and assumptions as well as the computational model and its verification and validation to the state of deformations, obtained by proper evaluation of the measured phenomena.

8. Conclusion

A critical review has been undertaken on mathematical and experimental methods applied in projecting engineering structures and in identifying the actual state of already existing objects. It turns out that both these methods are full of different sources of uncertainties, inaccuracies and errors. Therefore the question had been raised, whether and how to overcome the problem to avoid and/or to reduce such sources. As recommended by several authors a satisfactory or at least best possible answer can be seen in combining the mathematical and experimental tracks because of the coherence between both.

But certainly high demands are to make on the analysts. It is to consider as a strict presupposition that they have be able,

to know about the technical features, functions, operational conditions, requirements etc. concerning the structure in question,

to understand the background of the different mathematical processes and related computer-programs and to follow their coherences,

to understand the techniques of the respective experimental equipments and the way, how they work, especially if commercially available complex measurement systems including affiliated software are used.

In conclusion analysts have to be in charge of a broad spectrum of knowledge in different fields of engineering- and natural-sciences. In consequence this must be considered as a great challenge in engineering education.

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