

## Identification of parameters for models of ductile damage

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**Abstract:** This paper introduces the description of effective method of calibration of a material plasticity. This problem is solved in the project “Identification parameters of ductile damage materials for nuclear facilities”. The research focuses on the phenomenological material models and identification of their parameters. The calibration of the material parameter is based on the evaluation of the experimental samples series and FE simulations that are calculated in Abaqus 6.10 software.

**Keywords:** Plasticity, calibration, FEM, Johnson-Cook, ductile damage

### 1. Introduction

Because of the growing demands on safety, reliability and longer lifetime period of nuclear facilities components it is necessary to use material data in a numerical simulation that considers the ductile damage. The plastic strain represents one of the most crucial parts in the process of ductile damage.

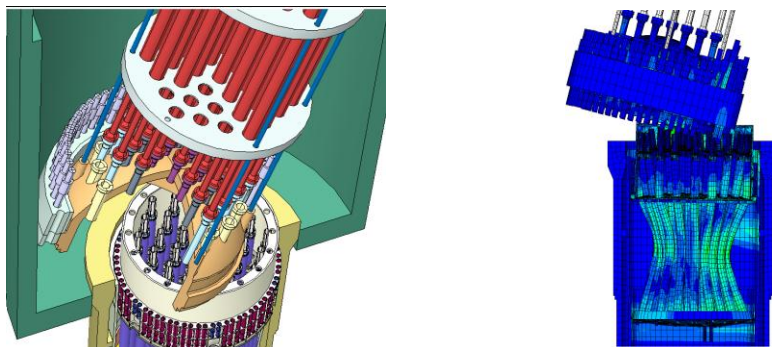


Fig. 1. Hypothetical fall study of top block in reactor shaft

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The ductile fracture occurs after all the plasticity reserve is consumed. If the damage additionally affects plastic behavior of the material then we can talk about the so called tied continual damage model. Von Mises plastic model and the isotropic hardening were used in this project. In this case it is necessary to calibrate dependence of actual yield stress on accumulated intensity of the plastic strain  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$ .

## 2. Calibration method of plasticity model

The calibration of material parameters is based on comparison of the measured response of the experimental samples and FE simulation results. The aim of calibration is to reach such values of material parameters so that the calculation characterizes experiment with the highest possible accuracy. For stated plastic model it is necessary to calibrate dependency of yield stress on accumulated intensity of plastic strain  $\sigma_Y^{True} = \sigma_Y^{True}(\varepsilon_{ln}^{pl})$ . The basic experimental test is uniaxial tensile test of smooth sample [1]. The stress and strain is described by the eq. (1)

$$\varepsilon = \frac{\Delta L}{L_0}, \quad \sigma = \frac{F}{S_0} \quad (1)$$

If the strain distribution is steady we can calculate the true stress and logarithmic strain from condition of constant volume using the eq. (2)

$$\varepsilon_{ln} = \ln(1 + \varepsilon), \quad \sigma^{True} = \sigma(1 + \varepsilon) \quad (2)$$

The next step is subtraction of elastic strain. We can determine this strain on initial yield stress from tensile diagram  $\sigma_Y$ .

$$\varepsilon_{ln}^{pl} = \varepsilon_{ln} - \varepsilon_{ln}^{el} = \varepsilon_{ln} - \frac{\sigma_Y}{E} \quad (3)$$

With this procedure the plastic part of tensile curve  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  can be determined until the time of local necking of sample when the eq. (2) aren't valid. The stress in the necking area isn't uniaxial anymore and plastic part of tensile curve needs to be determined by using the convenient correction (Bridgman) [4] or iteratively by using FE simulation. The necking of smooth sample which corresponds with extension of sample  $\Delta l_{neck}$  begins when the maximal force is reached through the experiment. It is possible to interpolate the general form of function  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  in suitably chosen points by sequence of corresponding points  $[(\varepsilon_{ln}^{pl})_j, (\sigma_Y^{True})_j]$ . The plastic part of tensile curve is entered in a form of table into FE programs. The final function is formed from two parts. The first one which is valid until creation of plastic necking  $\varepsilon_{ln}^{pl} < \varepsilon_{ln,neck}^{pl}$  is constituted by values of true stress and logarithmic strain which were calculated using the eq. (2), eq. (3) directly from the experimental data. The second one for  $\varepsilon_{ln}^{pl} \geq \varepsilon_{ln,neck}^{pl}$  is replaced by approximation function which is chosen to fulfil the condition of tangential connection in the area of plastic strain during the necking. The direction of tangent cannot be determined directly by the influence of

noise. Because of this the small area around critical plastic strain is approximated by quadratic function and direction  $\frac{d\sigma_Y^{True}(\varepsilon_{ln.neck}^{pl})}{d\varepsilon_{ln}^{pl}}$  is calculated analytically using this function. In this paper the plastic part of tensile curve was replaced by power function. This function corresponds to the first member of Johnson-Cook plastic model as it is shown in the eq. (4).

$$\sigma_Y^{True} = \left( A + B(\varepsilon_{ln}^{pl})^n \right) \left( 1 + c \ln \frac{\dot{\varepsilon}_{ln}^{pl}}{\dot{\varepsilon}_0^{pl}} \right) (1 - \tilde{T}^m) \quad (4)$$

where A, B, n, c, m are the material parameters. By application of tangential connection is obtained the equations system

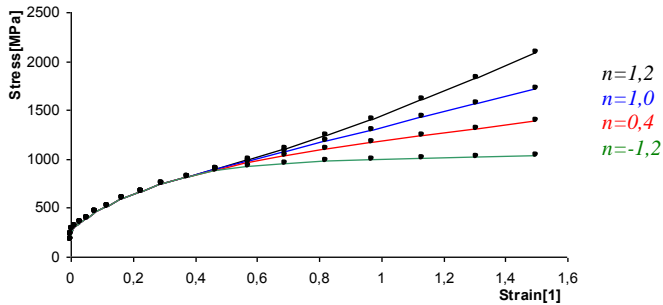
$$A + B(\varepsilon_{ln.neck}^{pl})^n = \sigma_{Y.neck}^{True} \quad (5)$$

$$Bn(\varepsilon_{ln.neck}^{pl})^{n-1} = \frac{d\sigma_Y^{True}(\varepsilon_{ln.neck}^{pl})}{d\varepsilon_{ln}^{pl}} \quad (6)$$

By solving the equations system the parameters A, B can be determined as a function of exponent n which can be calibrated iteratively. Because the approximate function is defined from value  $\varepsilon_{ln.neck}^{pl} > 0$ , the domain of exponent n can be extended into negative numbers. For nonzero direction of plastic part of tensile curve in the place of connection it is impossible to define approximation function for  $n = 0$  because it doesn't comply with the eq. (6) in general.

**Table 1. Properties of plastic model designed according to eq. (5), eq. (6) for different values n**

$n > 1$	Plastic strain-rate hardening after the necking of sample increases
$n = 1$	Plastic strain-rate hardening after the necking of sample is constant
$n < 1$	Plastic strain-rate hardening after the necking of sample decreases
$n = 0$	Model is not defined in general case
$n \rightarrow -\infty$	Plastic hardening after the necking is almost the ideal plastic material



**Fig. 2. Final shape of tensile plastic curve part for different n**

The final step of calibration is interpolation of both parts of dependency  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  into the properly chosen interpolation points. Distribution of these points should represent shape of plastic part of tensile curve. In the position of higher stress gradient it is suitable to increase the density of interpolation points. It is necessary to know that with an increasing number of the following parameters (temperature, plastic strain-rate) which define plastic model the total number of interpolation points will sharply increase.

$$(\varepsilon_{ln}^{pl})_j = a \cdot j^\alpha, j = 0, 1, 2, \dots, N, \alpha \geq 1 \quad (7)$$

where  $N$  is number of interpolation intervals. In most of the cases the ductile damage is simulated by using explicit solvers now. Material data are interpolated by given number of points with constant step. Using  $N \leq 50$  intervals defined by user, Abaqus uses  $100 \cdot N$  intervals with constant size for regularization. In case that the user defined interval is lower in some area than the size of regularized interval, the information about the course  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  is lost. Possible error could be eliminated by correct choice of maximal plastic strain and by suitable distribution of interpolation points. It is convenient that the lowest user defined interval equals size of regularized interval. This condition is described by the eq. (8)

$$a \cdot 1^\alpha = \frac{\varepsilon_{ln,max}^{pl}}{100 \cdot N} = \frac{a \cdot N^\alpha}{100 \cdot N}, \alpha \geq 1, \quad (8)$$

where  $\varepsilon_{ln,max}^{pl}$  is maximal defined plastic strain. The solution of equation is expression for calculation of exponent  $\alpha$  which is dependant only on number of chosen interpolation intervals.

$$\alpha = \frac{\ln(100N)}{\ln N}, \quad N \leq 50 \quad (9)$$

Parameter  $a$  is possible to calculate from condition of maximal presumptive strain.

$$a \cdot N^\alpha = \varepsilon_{ln,max}^{pl} \rightarrow a = \frac{\varepsilon_{ln,max}^{pl}}{N^\alpha} \quad (10)$$

This way the interpolation points could be distributed in dependence on presumptive range of reduced plastic strain and on number of chosen interpolation points.

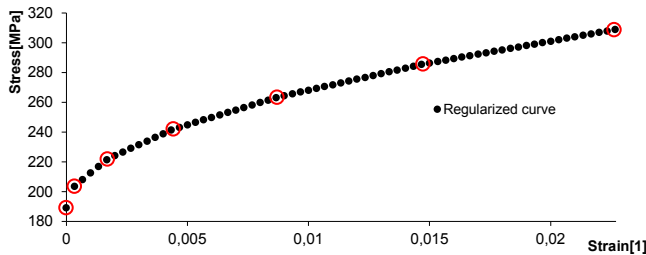


Fig. 3. Inception of regularized plastic function with suggested interpolation

### 3. Calibration of Johnson-Cook plastic model

The plastic part of tensile curve is often approximated by power dependence in practise in form that accords with the first member of Johnson-Cook plastic model [6] and it is implemented in software Abaqus. Using this approximation is in case of well sameness advantageous because the whole function  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  is described with only three parameters A, B and n. The function is continuous and the problem with the correct choice of interpolation points and problems with regularization of data if the material data is defined by table fall off. The condition of sameness experiment and material model in the point of necking can express parameters A, B in dependence of choosing parameter n. According to the article Havner, K. S. [3] to necking it attends if the function  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  complies the condition eq. (11)

$$\frac{d\sigma_Y^{True}(\varepsilon_{ln}^{pl})}{d\varepsilon_{ln}^{pl}} < \sigma_Y^{True}(\varepsilon_{ln}^{pl}) \quad (11)$$

Using the eq. (2) is it possible to derive special case of function that generates constant force response  $F_0$  during loading the sample.

$$\sigma_Y^{True}(\varepsilon_{ln}^{pl}) = \sigma_Y e^{\varepsilon_{ln}^{pl}} = \frac{F_0}{A_0} e^{\varepsilon_{ln}^{pl}} \quad (12)$$

where  $A_0$  is initial cross-section of testing sample. This function is the limit case of plastic necking origin in all range of plastic strain as the equation eq. (11) descripts. Substitute power dependence and application of condition force at necking for eq. (14) we became system of equations

$$A + B(\varepsilon_{ln.neck}^{pl})^n = Bn(\varepsilon_{ln.neck}^{pl})^{n-1} \quad (13)$$

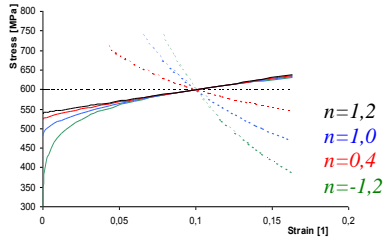
$$A + B(\varepsilon_{ln.neck}^{pl})^n = \sigma_{Y.neck}^{True} \quad (14)$$

The solution can be expressed parameters of plastic power dependence as the function of exponent n.

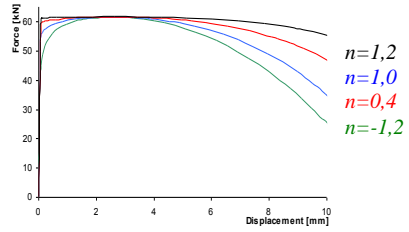
$$A(n) = \frac{\sigma_{Y.neck}^{True}(n - \varepsilon_{ln.neck}^{pl})}{n} \quad (15)$$

$$B(n) = \frac{\sigma_{Y.neck}^{True}(\varepsilon_{ln.neck}^{pl})^{1-n}}{n} \quad (16)$$

It is good reason to determine the parameter n iteratively. The function  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  isn't allowed to be negative in all range. From this reason must be positive  $A(n)$  which the physical meaning is initial yield stress  $\sigma_Y$ . The value of parameter  $n \geq \varepsilon_{ln.neck}^{pl}$ . In (Fig. 4) the course of power dependencies and their derivations for different n. The results of force response for simulated smooth sample are shown in (Fig. 5). The method described in this paper gives effective way to approximate the plastic response of material using this model.

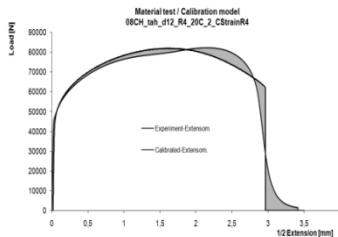


**Fig. 4.** Inception of regularized plastic function with suggested interpolation

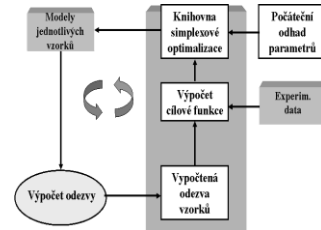


**Fig. 5.** Inception of regularized plastic function with suggested interpolation

Iterative calibration should be lengthy process for calibration more parameters. Therefore the optimizing script was writing in Python which is possible to run in Abaqus software. The surface between calculated and experimental measured curve of force-displacement is scale of accuracy as it is shown in (Fig. 6).



**Fig. 6.** Surface deviation between FE simulation and Experimental data



**Fig. 7.** Diagram of optimizing script for local optimization

The calibrating script is based on simplex algorithm for local optimization. This algorithm can optimize effectively more parameters at the same time. The disadvantage of local optimization is huge accuracy of estimate solution. Only the local minimum of target function is found using simplex algorithm. Own procedure is shown in (Fig. 7).

1. The initial estimations of parameters and experimental responses force-displacement for each sample are loaded into optimizing block. The database of FE models is created.
2. FE models of individual samples are modified in currently values of material data. The initially estimations are used in the beginning of calibration.
3. The response force-displacement is calculated and loaded into optimizing block for each sample.
4. The target function (size their deviation) is calculated from simulations and experiments. It is shown in (Fig. 6).
5. Optimizing algorithm designs useful changes of material data and the cycle 2 – 5 is repeated until the local minimum of target function isn't found.

#### 4. Calibration of plastic model temperature dependence

It is often required to calculate simulation for different temperatures of material in practice hence the plastic behaviour of material is calibrated with temperature dependence [6]. For some materials is possible to use proportional dependence  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  considering force response  $F(\Delta l)$ . If the measured dependencies force-displacement for fixed temperatures  $F(\Delta l)$  can be approximated well with dependence of force-displacement for the same sample for the reference temperature  $F(\Delta l)_{T_0}$  in form  $F(\Delta l)_T \approx \tau(T) \cdot F(\Delta l)_{T_0}$  where  $\tau(T)$  is convenient correcting function dependent on only temperature. After that the correcting function can be calculated as  $\sigma_{Y.crit}^{True}(\varepsilon_{ln}^{pl})_T = \tau(T) \cdot \sigma_{Y.crit}^{True}(\varepsilon_{ln}^{pl})_{T_0}$ . The correcting temperature function is designed in form

$$\tau(T) = 1 - \frac{T - T_0}{T_{melt} - T_0} \quad (17)$$

Where  $T_0$  is reference temperature of sample for which the plastic part of tensile curve was identified,  $T_{melt}$  is melt temperature of the material and  $m$  is material parameter which describes temperature softening. The own process of calibration can be divided into two steps. In the first step it is necessary to determine the value of correcting function for individual temperatures. The low square method is a convenient tool in this case. The correcting function described in form eq. (18) is using this method

$$\tau(T_j) = \frac{\int_0^{\Delta l} F(\Delta l)_{T_j} d\Delta l}{\int_0^{\Delta l} F(\Delta l)_{T_0} d\Delta l} \quad (18)$$

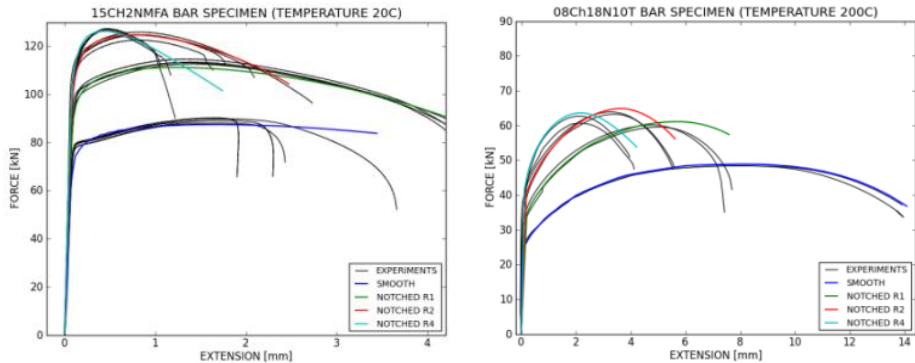
The second step is to find parameters  $m$  and  $T_{melt}$ . There is no way how to express the parameters therefore they are calculated by numerical simulation. This task is possible to convert to optimizing problem. The parameters are searched so that they minimize the next target function.

$$F(m, T_{melt}) = \sum_j |\tau_j - \tau(T_j)| \quad (19)$$

In cases that it is impossible to adjust the experimental dependence using convenient function in form  $F(u)_t \approx \tau(T) \cdot F(u)_{T_0}$  it is necessary to calibrate the dependence  $\sigma_Y^{True}(\varepsilon_{ln}^{pl})$  separately for each individual temperature.

#### 5. Results of calibration

The results of plastic calibration which were achieved by the method described in this paper are represented by material 08CH18N10T and 15CH2NMFA. The model of ductile damage wasn't used in this simulation. The plastic part of tensile curve of material 15CH2NMFA was approximated Johnson-Cook plastic model. The result of calibration is finding of parameters  $A$ ,  $B$ ,  $n$ ,  $T_{melt}$  and  $m$ . The (Fig 8) shows comparison of FE simulation of tensile tests for different temperature with experimental data.



**Fig. 8.** Comparison experimental dependence of force-displacement for tensile samples with different temperatures with FE simulations for materials 15CH2NMFA and 08CH18N10T

The plastic part of tensile curve of material 08CH18N10T was interpolated by a table. In this case it wasn't possible to approximate the temperature dependence with sufficient accuracy by corrective function  $\tau(T)$ . The plastic part of tensile curve was identified separately for all the individual temperatures.

## 6. Conclusion

This paper describes the method of calibration of the plastic part of a curve that performs an important aspect of ductile damage models. Basically two different ways of approximation of the dependence on actual yield stress and accumulated intensity of plastic strain were defined and an effective method for their identification was designed. This described technique is based on theoretical knowledge of the plastic material behaviour and hence substantially reduces demands of calibration. Identification of the independent parameters of the models is processed iteratively using an optimizing script that was developed as a part of this project. The calibration is largely corresponding with the experimental results as we can see in (Fig. 8). Currently the calibration of the plasticity models in dependence on the strain-rate is being designed and processed.

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