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Modulus of Elasticity Determination from Cantilever Deflection

Lubos Pazdera[1], Libor Topolar[2], Jaroslav Smutny[3] &Veronika Rodriguezova[4]

Abstract: The aim of laboratory experiment is to demonstrate different methods for cantilever deflection measuring and to simulate the finite element method and analytic calculation to students. A cantilever loaded in bending is an elementary example which demonstrates the possibility of measuring and simulating mechanical loads to students. In order to contain some practical output of the laboratory experiment, the cantilever is of square section shape and from a homogenous material. The cantilever bend is described by an analytical equation. The computing is simple and in principle quite clear if it is focused on a range of liner proportion (Hooke's law). It is possible to measure the strain by strain gauge and to record strain as near as possible to the fixation place. The deflection on the free end is measured similarly by a dial indicator. Another less known method is measuring by light mirror reflection. The deflection in different parts of the cantilever can be observed by a mathematical simulation, in this case by Finite Element Method. To achieve the right material properties, elements, boundary conditions and last but not least right meshing is necessary for a good simulation. The article includes the experiment itself and mathematical results.

Keywords: Experiment; Modulus of Elasticity, Elastic Modulus; Strain Gauge; Cantilever; Beam, Deflection; Mirror, Light, Finite Element Method, Teaching

1. Introduction

An elastic modulus or modulus of elasticity, is defined as the slope of its stress–strain curve in the elastic deformation region. [1]

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}} \tag{1}$$

Many types of modulus can be defined. Here are three basic ones: [2]

a) Young's modulus *E* which describes tensile elasticity

[1] *Prof. Lubos Pazdera,CSc.; Brno University of Technology, Faculty of Civil Engineering, Department of Physics, Veveri 331/95, 602 00 Brno, Czech Republic, pazdera.l@fce.vutbr.cz*

[2] *Mgr. Libor Topolář, Ph.D.; Brno University of Technology, Faculty of Civil Engineering, Department of Physics, Veveri 331/95, 602 00 Brno, Czech Republic, topolar.l@fce.vutbr.cz*

[3] *Prof. Dr. Jaroslav Smutny, Brno University of Technology, Faculty of Civil Engineering, Department of Railway Structures and Constructions, Veveri 331/95, 602 00 Brno, Czech Republic, smutny.j@fce.vutbr.cz*

[4] *;Mgr. Veronika Rodriquezova, Brno University of Technology, Faculty of Civil Engineering, Department of Social Sciences, Veveri 331/95, 602 00 Brno, Czech Republic, rodriguezova.v@fce.vutbr.cz*

b) shear modulus or modulus of rigidity G which describes an object's tendency to shear

c) bulk modulus K which describes volumetric elasticity

Young's modulus E can be calculated by dividing the tensile stress σ by the tensile strain ε in the elastic portion of the stress-strain curve. [3]

$$E = \frac{\sigma}{\varepsilon} \tag{2}$$

Young's modulus, the shear modulus, and the bulk modulus are related with help Poisson's ratio. When stretched in one direction, a material generally contracts in the other two directions. Poisson's ratio v is the negative of the ratio of the lateral or transverse strain ε_t to the axial strain ε in tensile loading, i.e. $\nu = -\varepsilon_t / \varepsilon$ [2]

Thus

$$E = 3 \cdot K \cdot (1 - 2 \cdot \nu) = 2 \cdot G \cdot (1 + 2 \cdot \nu) \tag{3}$$

Elasticity modulus belongs to basic material characteristics. It influences deformation material properties considerably and also consequently deformations of constructions as deflections, displacements, contractions etc. In generally, the greater the modulus, the smaller the deformations are and vice versa. The importance of elasticity modulus as one of the basic material characteristics increases with static intensity of the construction. [4,5]

2. Methods

Some different processes of detection Young's (elastic) modulus of rectangle cross section cantilever are demonstrated here. In this case the second moment of inertia, J, is defined by $J = b \cdot h^2 / 12$, where b is the width and h is the height of the beam cantilever according to Fig. 1. The deflection of the free end of the beam, y, and elastic modulus, E, are given by

$$y = \frac{F \cdot l^3}{3 \cdot E \cdot J} \quad \Rightarrow \quad E = \frac{4 \cdot l^3 \cdot g}{b \cdot h^3} \cdot \frac{m}{y} \tag{4}$$

where F is force, m, is the weight, l is the length of the beam, or more precisely the distance of the force from the fixing, and g is gravitational acceleration. It is possible to detect the dependence of the deflection on the loading by inductive displacement transducer.

Bending stress, σ, is given by [6]

$$\sigma = \frac{M_o}{W_o} = \frac{F \cdot l}{\frac{1}{6} \cdot b \cdot h^2} \tag{5}$$

where M_o is the bending moment, W_o is the bending section modulus and F is the force making moment. By applying (5) and (2) for elasticity modulus, E, is given

$$E = \frac{6 \cdot l \cdot g}{b \cdot h^2} \cdot \frac{m}{\varepsilon} \qquad \Rightarrow \qquad E = \frac{6 \cdot a \cdot g}{b \cdot h^2} \cdot \frac{m}{\varepsilon} \tag{6}$$

The dependence of the relative strain on the load can be obtained by sticking the strain gauge on the beam surface to the fixing as close as possible, because the maximal strain is expected here (Fig. 2).

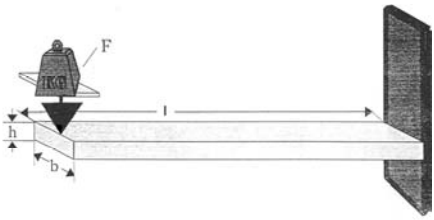

Fig. 1. Beam cantilever

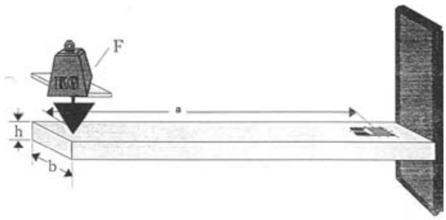

Fig. 2. Beam cantilever with strain gauge

Similarly, the slope at the free end of the beam, α, and elasticity modulus, E, are [7]

$$\alpha = \frac{F \cdot l^2}{2 \cdot E \cdot J} \quad \Rightarrow \quad E = \frac{6 \cdot l^2 \cdot g}{b \cdot h^3} \cdot \frac{m}{\alpha} \tag{7}$$

The mirror method could be used advantageously for measuring the slope as shown in Figs. 3 and 4. The mirror method principle is sketched in Fig. 3. A light beam dispatched from point n_0 after the mirror reflection is detected at point n. Sloping the mirror at an angle α causes a change of the light beam direction reflected from the mirror by an angle 2α. The scale is placed perpendicularly to the light beam reflected from equilibrium mirror position (see Fig. 3 dashed). The light beam is detected in point n_0. The light beam hits point n when the mirror is sloped at an angle α. For very small angles, α, the approximate equation can be used (Fig. 3). [8]

$$\tan(2 \cdot \alpha) = \frac{n - n_0}{L} \quad \rightarrow \quad \alpha < 5^0 \quad \Rightarrow \quad \alpha \approx \frac{n - n_0}{2 \cdot L} = \frac{\Delta n}{2 \cdot L} \tag{8}$$

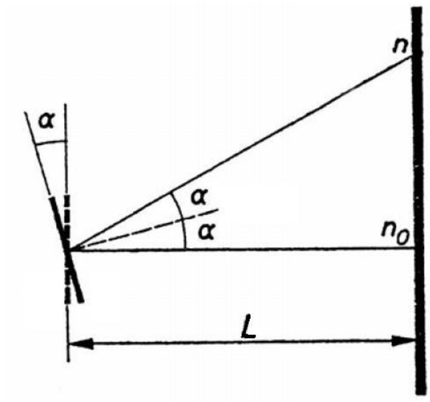

Fig. 3. Angle measuring by a mirror

Fig. 3 and equation (8) show that the further the distance from the mirror, L, to the scale, the more precise the angle α.

Thus (7) and (8) imply the elasticity modulus, E,

$$E = \frac{12 \cdot l^2 \cdot g \cdot L}{b \cdot h^3 \cdot \Delta n} \cdot \frac{m}{\alpha} \tag{9}$$

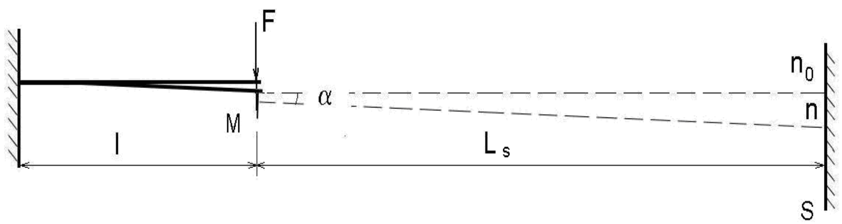

Fig. 4. Using a mirror to measure deflection of the cantilever (M – mirror, S – scale, F – force, l, L_s – length, α – angle, n, n_0 – units on the scale)

A laboratory practice or a lecture experiment, which shows the possibilities of applications of eqs. (4), (6) and (9), can be easily implemented for instance by following the Figs. 5 and 6.

Fig. 5. Experimental set up

Fig. 6. Indication of laser ray movement

3. Results

A steel cantilever beam of 3.5 mm in height and 25 mm in width was chosen for the laboratory experiment. It was fastened to the fixation at the length of 325 mm. On the top and button face of the cantilever, two strain gauges at 300 mm from the free end were glued to measure the strain. The free end of the cantilever was loaded by weights increasing by 0.1 kg up to 1 kg. The system SPIDER or QUANTUM respectively, was applied for measurements of strain by strain gauge and deflection by inductive displacement transducers. Measurements of the slope or the angle were realized either by a simple method when a laser pointer was fixed on the free end of the cantilever or by the mirror method when a laser pointer was fixed to the scale and a small mirror was glued on the free end of the cantilever. A simple scale is shown in Fig. 6.

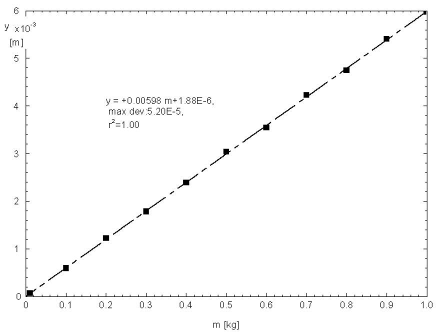

Fig. 7. Cantilever deflection (y – deflection, m – load)

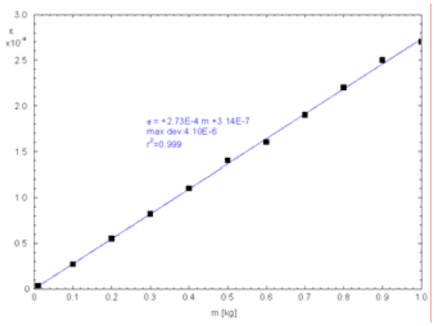

Fig. 8. Cantilever strain (ε – strain, m – load)

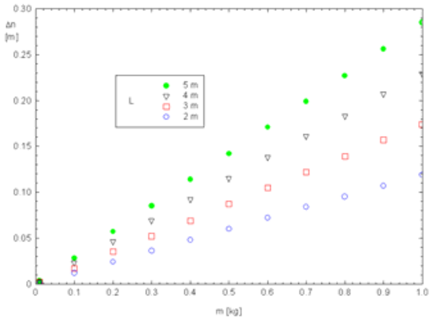

Fig. 9. Ray of light indication (Δn – difference of light, m – load)

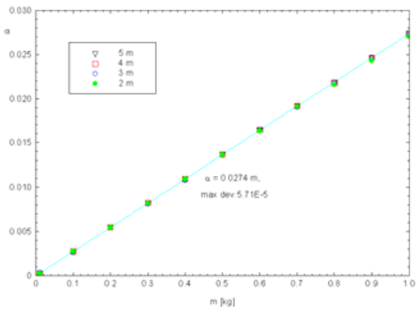

Fig. 10. Slope computed by means of a mirror (α – angle, m – load)

Eqs. (4), (6) and (9) contain elements $\frac{y}{m}$, $\frac{\varepsilon}{m}$ a $\frac{\alpha}{m}$, which are determined by the slopes of the measured curves $y(m)$, $\varepsilon(m)$ a $\alpha(m)$ according to charts in Figs. 7, 8 and 10. The behaviours of all dependencies are linear, so the least squares method can be implemented.

However, the mirror method accuracy will be dependent on the relation between the length of the cantilever, i.e. the position of the mirror on the cantilever, and the distance of the mirror from the scale as well as on the slope of the cantilever. Fig. 9 shows clearly that with the increasing distance between the mirror and the scale L, the recorded value of Δn also increases and consequently the relative accuracy of the reading increases as well. Hence from (9) and Fig. 4, the length is approximately $L \cong l/2 + L_s$.

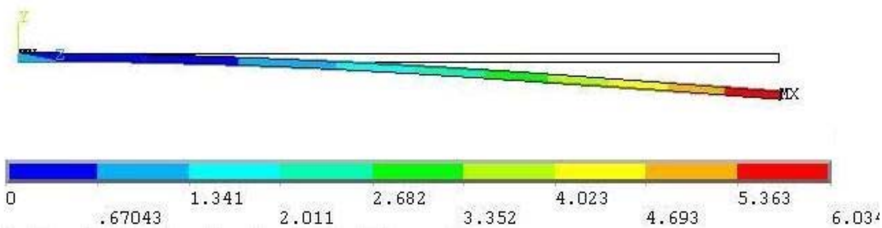

Fig. 11. Cantilever deflection

The comparison of real measurement results and mathematical modelling results carried out by Finite Element Method computed by software ANSYS shows a significant accordance. Fig. 11 shows deflection at vertical direction in different parts of the cantilever at 1 kg loading at the free end. Distribution of longitudinal strain on the cantilever surface is demonstrated in Fig. 12; it shows advantages of placing the strain gauge near the fixing area, where the changes of strain and stress are maximal. The strain in the cross section area, where the strain gauge is placed, is shown in Fig. 13. The maximum principal strain or compression is in correspondence of theoretical assumptions on the surface of the cantilever. [9,10]

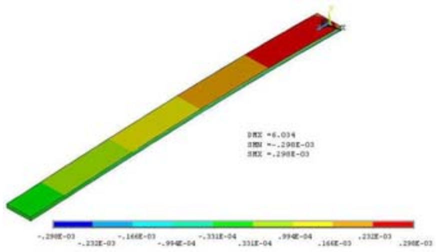

Fig. 12. Cantilever longitudinal strain.

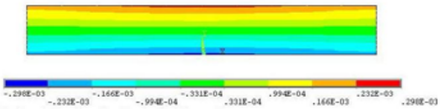

Fig. 13. Distribution of strain in the cross section where the strain gauge is placed

4. Conclusions

The aim of the article is to show possibilities of acquiring Young's modulus for a cantilever by three experimental methods. Measurement equipment for detecting the deflection and slope are relatively uncomplicated. The strain measurements by strain gauge are a bit more demanding from both economical and technical aspects. There are steel cantilever moduli of elasticity measured by three processes (measurement of deflection, strain and slope) in Tab. 1. Measurement accuracy is better than 10 % which is adequate for most of the real experiments.

Table 1. Recommended mathematical element sizes

equation	Young's modulus E [$\times 10^9$ Pa]
y (4)	210
α (6)	212
ε (9)	211

The application of the mirror method, when compared with common methods, proves as an easy and interesting technique for a rough determination of modulus elasticity of cantilever.

A connection of a simple and a sophisticated experiment with a mathematical numeric simulation could contribute to the better understanding of cantilever behaviour from the point of stress and strain analysis, when these methods are used in laboratory practice by students.

Acknowledgements

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