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Improved Method for Identification of LCF Parameters

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Abstract: The conclusion of my contribution on past EAN Conference was that an attempt to identify low cycle fatigue (LCF) parameters of material from a shape of its hysteresis loop failed due to invalidity of Massing's rule on similarity of stress cyclic curve with a shape of hysteresis loop. The contribution improves the classical way of finding LCF parameters by complementing Basquin-Manson-Coffin equations by Ramberg-Osgood equation in an identification process.

Keywords: Low cycle fatigue; Parameters; Identificastion

1. Introduction

Behavior of materials under cyclic loading is described by Basquin [1] and Manson-Coffin [2, 3] equations. The equations contain four independent material parameters entering into relations between amplitudes and stresses, or plastic strains and fatigue lives, respectively. The low cycle fatigue parameters are obtained by statistical processing of data gathered during fatigue tests of a particular material. The classical procedure of the processing can be found in standards (see [6, 7, 8] for instance) built forty years ago. They are based on a manual recording of measured data into tables and their processing by using office calculators.

At the present time, the situation has changed significantly, not only in measurement possibilities, but also in processing means. The following parts of the contribution deal with the state of the art of the problem.

2. Identification of LCF parameters

Fatigue properties of matarials in the elastic domain are expressed by equation [1]:

$$\sigma_a = \sigma_f' \left(2N_f\right)^b, \tag{1}$$

in which
σ_a is an amplitude of harmonic stress,
σ_f' is a coefficient of cyclic fatigue strength,
N_f is a number of load cycles to failure, and
b is an exponent of cyclic fatigue stress.

The equation was found more than 100 years ago by Basquin [1]. More than 40 years later, professors Manson [2] and Coffin [3] discovered that the similar equation held for plastic strain:

$$\varepsilon_{ap} = \varepsilon_f' \left(2N\right)^c, \tag{2}$$

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where ε_{ap} is an amplitude of plastic strain,
ε'_f is a coefficient of cyclic fatigue ductility, and
c is an exponent of cyclic fatigue ductility.

Equations (1) and (2) constitute a base for classical way of statistical processing of measured data represented by a set of triples $[\sigma_a,\, \varepsilon_{ap},\, N_f]_j,\ j = 1, 2,\, ...,\, J$, where J is a number of tested specimens. Both stress and strain amplitudes were measured manually on drawings of hysteresis loops recorded during the tests by x-y plotter in past. Nowadays, the processes $\sigma(t)$ and $\varepsilon(t)$ are digitized during the test and vectors of both signals samples are stored in computer memory for later processing.

2.1. Classical method

Both equations can be transformed into linear form by application of decimal logarithms to vectors of amplitudes and fatigue lives, all of the dimension J:

$$\begin{aligned} \log \boldsymbol{\sigma_a} &= \log \sigma'_f + b \log(2\boldsymbol{N_f}) \\ \log \boldsymbol{\varepsilon_{ap}} &= \log \varepsilon'_f + c \log(2\boldsymbol{N_f}), \end{aligned} \tag{3}$$

or rewritten into the matrix form

$$\underbrace{\begin{bmatrix} \log\,(2\,\boldsymbol{N}_f)_1 & 1 \\ \vdots & \vdots \\ \log\,(2\,\boldsymbol{N}_f)_J & 1 \end{bmatrix}}_{\boldsymbol{A}} \times \underbrace{\begin{bmatrix} b & c \\ \log \sigma'_f & \log \varepsilon'_f \end{bmatrix}}_{\boldsymbol{C}} = \underbrace{\begin{bmatrix} \log \boldsymbol{\sigma}_{a,1} & \log \boldsymbol{\varepsilon}_{ap,1} \\ \vdots & \vdots \\ \log \boldsymbol{\sigma}_{a,J} & \log \boldsymbol{\varepsilon}_{ap,J} \end{bmatrix}}_{\boldsymbol{B}} . \tag{4}$$

The system of equations (4) is over-determined, because the standards require at least 8 specimen for one material test while there are only two pairs of unknowns in $\boldsymbol{C}$. In consequence of inaccuracies of the model (3) and measurements, there is no exact solution of the system (4). Any matrix $\boldsymbol{C}$ of parameters generates a nonzero matrix of residuals

$$\boldsymbol{R} = \boldsymbol{A}\,\boldsymbol{C} - \boldsymbol{B}\,. \tag{5}$$

However, there might be such matrix $\boldsymbol{C}$, for which a norm of the residual matrix $\boldsymbol{R}$ becomes minimum. The common routine requires the Eucleidian norms of column vectors $\boldsymbol{r}_j,\ j = 1, 2, ..., J$ being minimum, what is equivalent with the condition

$$\boldsymbol{r}_j^T \boldsymbol{r}_j = \sum_{\forall j} r_j^2 \longrightarrow \text{minimum}, \tag{6}$$

where the superscript T designates the matrix transposition. The last equation is a base for the popular method of least squares, which is also used in the standard [8]. The solution corresponding to condition (6) may be obtained in the following steps:

- Multiply equation (4) by $\boldsymbol{A}^T$, the transposed matrix of $\boldsymbol{A}$, from left:

$$\boldsymbol{A}^T \boldsymbol{A}\,\boldsymbol{C} = \boldsymbol{A}^T \boldsymbol{B}. \tag{7}$$

- Calculate unknowns in matrix $\boldsymbol{C}$:

$$\boldsymbol{C} = \underbrace{(\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T}_{\boldsymbol{A}^+} \boldsymbol{B}. \tag{8}$$

Matrix $\boldsymbol{C}$ yields the best approximation of equation (3). Matrix $\boldsymbol{A}^+$ is called Moore-Penrose pseudoinverse of matrix $\boldsymbol{A}$. As soon as the matrix $\boldsymbol{C}$ be known, the parameters σ'_f and ε'_f are evaluated as

$$[\sigma'_f,\ \varepsilon'_f] = 10^{\,[\log \sigma'_f,\ \log \varepsilon'_f]}\,. \tag{9}$$

2.2. Improved method

Low cycle fatigue parameters $[\sigma'_f, b, \varepsilon'_f, c]$ obtained by classical method fit good only fatigue lives. Unfortunately, fatigue lives are the most problematic quantities. Are laboratories accepting fatigue life as a number of cycles N_f at a total break down of the specimen. The other use for N_f the number of cycles, for which the amplitude of stress drops down by 25%. We decided to chose such a number of cycles N_f for which stress amplitude becomes *unstable*. Till that moment, the specimen is in a steady state with constant properties. As soon as amplitudes of stress start to vary, the specimen has changed its properties. It would not be fair to evaluate material properties from an object different from that at the beginning of the test.

Fatigue lives of specimens are not the only observed quantities during tests. The recorded processes of stresses and strains enable to measure also turning points of hysteresis loops. These points are laying on the cyclic stress-strain curve, which is also a function of LCF parameters. It is obvious that after elimination of terms $(2N_f)$ from equations (1) and (2), the equation of Ramberg and Osgood is born:

$$\sigma_a = K'\,\varepsilon_{ap}{}^{n'}\,, \tag{10}$$

where $n' = \dfrac{b}{c}$ is cyclic strain hardening exponent, and

$K' = \dfrac{\sigma'_f}{\varepsilon'_f{}^{n'}}$ is cyclic stress hardening coefficient.

The formulae for n' and K' contain the same unknown LCF parameters like equations (1) and (2), what means that they may be included into the identification process. As a result of this step, the new identified LCF parameters fit both fatigue lines and cyclic stress-strain curve in the sense of minimum sum of squared residuals.

There is one weak point in the just described procedure. The Manson-Coffin equation (2) is based on an amplitude of *plastic* strain, which is not measured. Only total strain amplitude is controlled and measured during hard loading. It means that ε_{ap} should be evaluated as a difference between total and elastic strains

$$\varepsilon_{ap} = \varepsilon_{at} - \varepsilon_{ae} = \varepsilon_{at} - \frac{\sigma_a}{E'}\,. \tag{11}$$

A new quantity, cyclic Young's modulus E', comes into account. There are following questions: Is its value independent on specimen loading history? Is its static value E constant during test? Experimental investigations reveal that slopes E' of steady hysteresis curves are different from the static E obtained from tensile tests. The cyclic modulus E' used to be lower than the static E by tens of percents, what significantly influences the resulting ε_{ap}. Since the exact value of E' is unknown, it may be included among the unknown LCF parameters to be identified.

The sought parameters, to be identified, are

$$p = [\,\sigma_f',\, b,\, \varepsilon_f',\, c,\, E'\,]^T \quad \text{or} \quad [\,\sigma_f',\, b,\, \varepsilon_f',\, c\,]^T\,, \tag{12}$$

if modulus E' be known. The evaluation of residuals, preceded by an evaluation of n' K' and ε_{ap} in each iteration step, is performed in logarithmic domain just like in case of the classical approach:

$$\begin{array}{lcll}
\boldsymbol{r}_\sigma & = & \log(\sigma_f'\,(2\boldsymbol{N_f})^b) - \log \boldsymbol{\sigma_a} & : \ \text{Basquin} \\
\boldsymbol{r}_{MC} & = & \log(\varepsilon_f'\,(2\boldsymbol{N_f})^c) - \log \boldsymbol{\varepsilon_{ap}} & : \ \text{Manson} - \text{Coffin} \\
\boldsymbol{r}_{RO} & = & \log(K'\,\boldsymbol{\varepsilon_{ap}}^{n'}) - \log \boldsymbol{\sigma_a} & : \ \text{Romberg} - \text{Osgood} \\
\boldsymbol{r}_\varepsilon & = & \log\left(\left(\dfrac{\boldsymbol{\sigma_a}}{K'}\right)^{c/b} + \dfrac{\boldsymbol{\sigma_a}}{E'}\right) - \log \boldsymbol{\varepsilon_{at}} & : \ \text{"}\,\varepsilon_{ap} + \varepsilon_{ae} - \varepsilon_{at}\,\text{"} \\
r_E & = & (E' > E) * (E' - E) & : \ \text{a penalty.}
\end{array} \tag{13}$$

If there is n valid specimens for processing, the column vector of residuals used by the optimization procedure is of dimension $4n + 1$ and in the form

$$\boldsymbol{r} = \left[\,\boldsymbol{r}_\sigma^T, \boldsymbol{r}_{MC}^T, \boldsymbol{r}_{RO}^T, \boldsymbol{r}_\varepsilon^T, r_E\,\right]^T. \tag{14}$$

Only the last element of vector $\boldsymbol{r}$ is a scalar. In fact, it is no residual, but a penalty for the situation that E' tries to overstep a value of the static modulus E during the optimization. In consequence of it, the value of E' reaches the value E at maximum.

It is obvious that all formulae but the first one are nonlinear in parameters, what causes that a nonlinear solver should be used. The program **LCF** built in MATLAB applies Levenberg-Marquardt algorithm in Fletcher's version [13] for the purpose. The program enables to read several kinds of data:

1. Tables of experimental data prepared manually with columns [$\boldsymbol{\sigma_a},\ \boldsymbol{\varepsilon_{at}},\ \boldsymbol{N_f}$].
2. Text files containing 3-column tables of relative sampling times $\boldsymbol{t}$, loading forces $\boldsymbol{F}(t)$ and absolute elongations $\Delta\boldsymbol{L}(t)$ of an extensometer base.

3. Experiments and data processing

Low cycle fatigue experiments were performed in the year 2010 on specimens made out of carbon steel ČSN 41 1523.1 and documented in the report [11]. The experiment settings and measured fatigue lives were gathered in Table 1 taken from protocol [11], where the experiment is described. A processing of the measured data is presented in an extended standard form [8] in report [12].

Table 1. Data of low cycle fatigue tests of ČSN 41 1523.1

Spec.#	σ_a [MPa]	ε_{at} [-]	N_f [-]
1	426	0.012058	540
2	421	0.011288	860
3	230	0.001892	76000
4	282	0.003789	14171
5	393	0.00785973	1443
7	246	0.00212822	60560
9	434	0.01172978	520

The program **LCF** can process several kinds of data into material LCF parameters by fitting measured fatigue lives as described in the following subsections.

3.1. Tabular approach

The simplest way of measured data processing consists in the classical method dealt in subsection 2.1 applied to data from Table 1.The found parameters can be taken as a good initial approximation for the nonlinear regression under subsection 2.2. If the measured data show low scatter, the resulting graphs are almost identical as seen in Figure 1 and Figure 2.

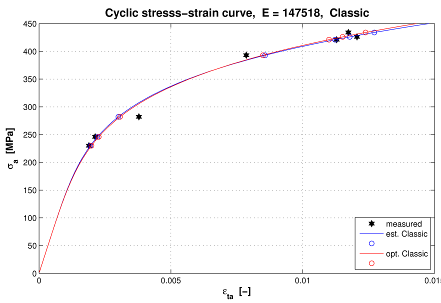

Fig. 1. Cyclic stress-strain curve

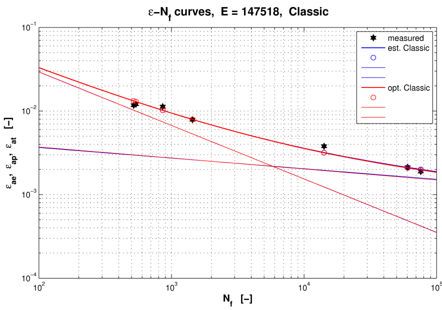

Fig. 2. ε_{ap}-N_f curve

The initial estimate of LCF parameters for nonlinear regression need not be only evaluated by linear regression. They may be estimated also under empirical formulae, say by Bäumler and Seeger (B&S), presented in [14]. The resulting graphs are shown in Figure 3 and Figure 4.

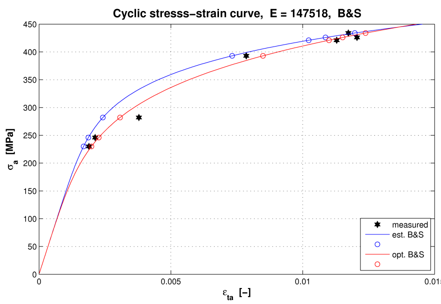

Fig. 3. Cyclic stress-strain curve

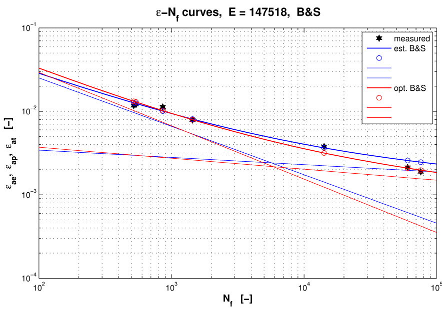

Fig. 4. ε_{ap}-N_f curve

Another attempt for estimation of LCF parameters has been made with parameters taken from the unsuccessful identification from shapes of hysteresis loops discussed in EAN 2011 [10]. It is obvious from the Figure 5 that new cyclic stress-strain curve almost coincides with the old one, in spite of completely different parameters. This phenomenon has been caused by the fact that points of cyclic curves also belong to hysteresis loops from which the initial parameters were identified. However, serious differences are evident in Figure 6.

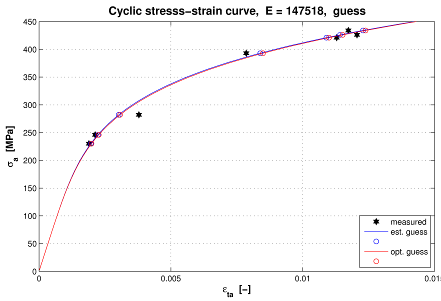

Fig. 5. Cyclic stress-strain curve

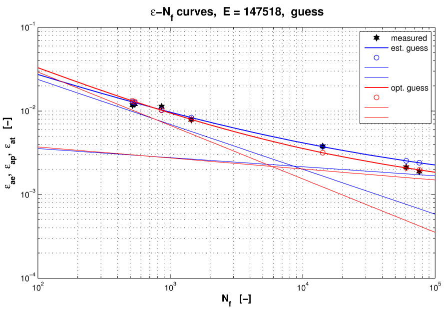

Fig. 6. ε_{ap}-N_f curve

In all examples, modulus E' obtained from the slopes of hysteresis loops was used. As a known parameter, E' did not enter the identification. If the static modulus E be accepted, the identified LCF parameters would be different. Experiment in identification revealed that the identified value of E' would often be greater than that of static E what is unrealistic. This was a reason why the vector of residuals was complemented by the penalty described above.

All estimated and identified values of the LCF parameters from the upper examples are gathered in Table 2. The second column marked as 'LR' belongs to the classical (linear regression) method that respects only Basquin and Manson-Coffin equations. When this result or any other set of parameters were used as an initial estimate, the nonlinear regression respecting also Romberg-Osgood equation (10) returned practically equal results as seen in columns marked by 'NLR'. There are no significant differences between lines for LR and NLR identification in Figures 1 and 2 in spite of almost 5% differences among coefficients.

Table 2. A survey of estimated and optimized LCF parameters, $E' = 147518$ [MPa]

par	LR	NLR	B&S	B&S NLR	[10]	[10] NLR
σ'_f	1052.90	1099.52	802.50	1099.52	933.20	1099.53
b	-0.1263	-0.1315	-0.0870	-0.1315	-0.1087	-0.1315
ε'_f	0.8735	0.8657	0.5438	0.8657	0.4104	0.8657
c	-0.6402	-0.6391	-0.5800	-0.6391	-0.5370	-0.6391
n'	0.1973	0.2057	0.1500	0.2057	0.2024	0.2057
K'	1081.37	1132.63	879.28	1132.63	1117.56	1132.63

3.2. Sampled signals approach

There are big differences between ways of measurement in past and nowadays. While the old measurements have generated plots of hysteresis loops that have been processed manually off line, up to date methods sample signals of loading forces and deformations and store corresponding time series of samples in computer memories. They are later evaluated into vectors of samples of stress $\boldsymbol{\sigma}(kT)$ and strain $\boldsymbol{\varepsilon}(kT)$, respectively, where k is an order number of the sample and T a sampling period.

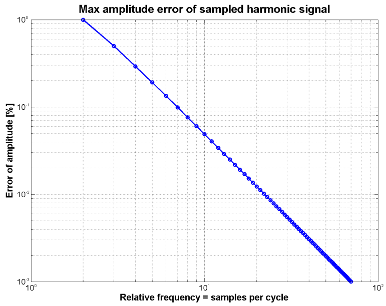

Fig. 7. Maximum amplitude error

The sampling frequency $f_s = 1/T$ should be chosen carefully in order that extremes of signals were found with an error not bigger than an permitted level. The well known criterion of Shanon-Kotelnikov declares that any signal can be reconstructed from its samples, provided a sampling frequency be at least twice a signal frequency. This rule may be used for long signals. If only one cycle is at disposal, there might be big differences between maximum sample value and a real cycle amplitude. Figure 7 shows the maximum error attainable under a relative sampling frequency. The error of about one percent is gained for sampling frequencies more than 20 times the signal frequency.

Drawbacks of the classic method under the standards [6]-[8] are, that only a single hysteresis loop is taken from about a halve of the total fatigue life under current test for en evaluation of LCF parameters. Any measurement deviation may influence results unpredictably. The similar effect can have the badly defined fatigue life N_f. In order to diminish those errors, new rules have been accepted:

- The total length of a recorded time series is reduced to a steady part. the first cycles do not come into evaluations but the number of cycles. As soon as the stress amplitude becomes instable, the rest of the time series is discarded.
- The selected steady interval of the time series is split into small subintervals for evaluation of stress and strain amplitudes.
- The average amplitudes are then evaluated and put into the table for the later evaluation of LCF parameters by any method used in section 2.

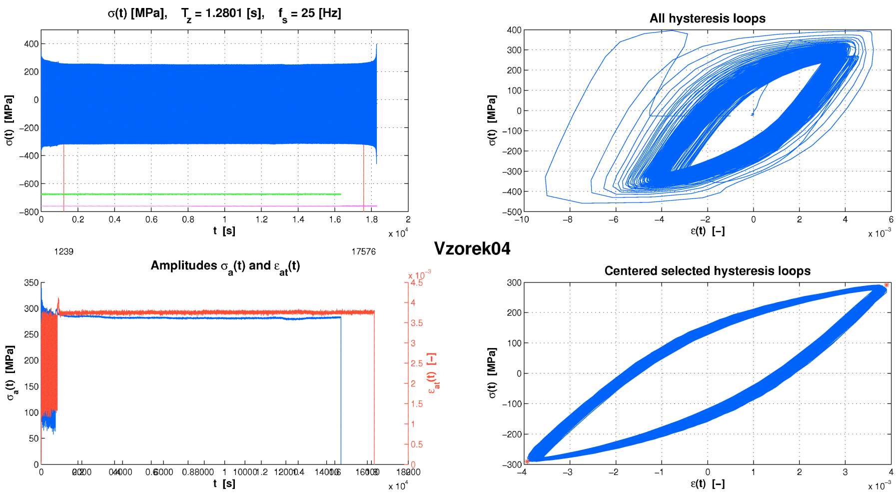

Fig. 8. Loading and processing of specimen #4

Figure 8 displays data measured during a test of specimen #4, all hysteresis loops, those loops from a selected steady interval and evaluated stress and strain amplitudes.

4. Conclusions

The contribution introduced an alternative way of identification of material LCF parameters, if compared to the standard one. It promises to be better and more reliable than the current methods. The procedures are based both on linear and nonlinear regression. The linear regression is formulated in a new way and should give just the same results as the standard method. The nonlinear regression is used when both Basquin-Manson-Coffin equations and Romberg-Osgood equations should be fulfilled.

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