EAN 2012
50th annual conference on experimental stress analysis

Load of fibres driving an inverted pendulum system

Pavel Polach[1], Jaroslav Václavík[2] & Michal Hajžman[3]

Abstract: One of interesting applications of cables or fibres is replacement of the chosen rigid elements of manipulators or mechanisms by those flexible elements. The main advantage of this design is the achievement of a lower moving inertia, which leads to a higher mechanism speed and lower production costs. An inverted pendulum attached to a frame by two fibres serves as a typical testing system for the investigation of the fibres properties influence on the system dynamic response. Motion of the pendulum of this nonlinear system is investigated using the **alaska** simulation tool and an in-house program created in the MATLAB system. The influence of some parameters of the system of inverted pendulum driven by fibres has already been investigated. In this paper the check of not exceeding the permissible load of fibres during their acting under dynamic loading is given. From the tested fibres the "light" fibre made of thin carbon fibres at Czech Technical University in Prague is considered for the checking. Maximum permissible load was measured at the University of West Bohemia. Due to the "light" fibre weight the massless fibre model is considered.

Keywords: Inverted pendulum; Fibres; Vibration

1. Introduction

One of interesting applications of cables or fibres is the replacement of the chosen rigid elements of manipulators or mechanisms by those flexible elements [1]. The main advantage of this design is the achievement of a lower moving inertia, which can lead to a higher machine speed and lower production costs. Drawbacks can be associated with the fact that cables should be only in tension [2,3] in the course of a motion. The possible cable modelling approaches should be tested and their suitability verified in order to create efficient mathematical models of cable-based manipulators mainly intended for the control algorithm design. An inverted pendulum driven by two fibres attached to a frame (see Fig. 1) is a simplified representation of a typical cable manipulator. The motion of the pendulum of this nonlinear system is investigated using the **alaska** simulation tool and using an in-house software created in the MATLAB system. The influence of certain parameters of the system of inverted pendulum driven by fibres has already been

[1] *Dr. Ing. Pavel Polach; Section of Materials and Mechanical Engineering Research, Výzkumný a zkušební ústav Plzeň s.r.o.; Tylova 1581/46, 301 00, Czech Republic; polach@vzuplzen.cz*
[2] *Ing. Jaroslav Václavík; Dynamical Testing Laboratory, Section of Testing Laboratories, Výzkumný a zkušební ústav Plzeň s.r.o.; Tylova 1581/46, 301 00, Czech Republic; vaclavik@vzuplzen.cz*
[3] *Ing. Michal Hajžman, Ph.D.; Department of Computer-Aided Modelling, Section of Materials and Mechanical Engineering Research, Výzkumný a zkušební ústav Plzeň s.r.o.; Tylova 1581/46, 301 00, Czech Republic; hajzman@vzuplzen.cz*

investigated. The influence of the actuated fibres motion on the pendulum motion in the case of their simultaneous harmonic excitation was investigated in [4], in the case of non-symmetric harmonic excitation it was investigated in [5]. The effect of the fibres preload on the pendulum motion was investigated in [6], the effect of the fibres' mass on the pendulum motion was investigated in [7]. Validation of permissible load of fibres during their acting under dynamic loading (on the basis of research report [8]) is checked in this paper. Simultaneous harmonic excitation is considered.

2. Possibilities of the cable modelling

The cable (fibre, wire etc.) modelling [9] should be based on considering the cable flexibility and the suitable approaches can be based on the flexible multibody dynamics (see [10]). The simplest way how to incorporate cables in equations of motion of a mechanism is the force representation of a cable (e.g. [11]). It is assumed that the mass of cables is small to such an extent comparing the other moving parts that the inertia of cables is negligible with respect to the other parts. The cable is represented by the force dependent on the cable deformation and its stiffness and damping properties. This way of the cable modelling is probably the most frequently used model in the cable-driven robot dynamics and control.

A more precise approach is based on the representation of the cable by a point-mass model (e.g. [12]). The cable can be considered either flexible or rigid. It has the advantage of a lumped point-mass model. The point masses can be connected by forces or constraints.

In order to represent bending behaviour of cables their discretization using the finite segment method [10] or so called rigid finite elements [13] is possible. Standard multibody codes (SIMPACK, MSC.ADAMS, **alaska** etc.) can be used for this purpose. Other more complex approaches can utilize nonlinear three-dimensional finite elements [14] or can employ the absolute nodal coordinate formulation (ANCF) elements [10].

From the tested fibres the "light" fibre made of thin carbon fibres is considered for the checking of not exceeding the permissible load of fibres during their acting under dynamic loading. On the basis of the results in [4,7,16] in the system of inverted pendulum the massless model of fibres is used.

3. Inverted pendulum

Already mentioned inverted pendulum, which is attached and driven by two fibres and affected by a gravitation force, was chosen as an example of the investigation of fibres' behaviour – see Fig. 1 (e.g. [4]). When the pendulum is displaced from the equilibrium position (i.e. "upper" position) it is returned back to the equilibrium position by the tightened fibre.

The model of the system of the inverted pendulum is considered to be two-dimensional. The system kinematics can be described by angle φ (one degree of freedom) and prescribed kinematic excitation $x(t)$. The equation of motion is of the form

$$\ddot{\varphi} = \frac{1}{I_A}\left(F_{v1} d \sin\alpha_1 - F_{v2} d \sin\alpha_2 + mg\frac{l}{2}\sin\varphi \right) \tag{1}$$

where I_A is the moment of inertia of the pendulum with respect to the axis in point A (see Fig. 1), α_1 and α_2 are angles between the pendulum and the fibres, m is the mass of the pendulum, g is the gravity acceleration and l is the length of the pendulum. The forces acting on the pendulum from the fibre

$$\begin{aligned} F_{v1} &= \left[k_v\left(l_{v1} - l_{v0}\right) + b_v \frac{\mathrm{d}l_{v1}}{\mathrm{d}t} \right] \cdot H\left(l_{v1} - l_{v0}\right), \\ F_{v2} &= \left[k_v\left(l_{v2} - l_{v0}\right) + b_v \frac{\mathrm{d}l_{v2}}{\mathrm{d}t} \right] \cdot H\left(l_{v2} - l_{v0}\right), \end{aligned} \tag{2}$$

where k_v is the fibre stiffness, b_v is the fibre damping coefficient and $H(\cdot)$ is the Heaviside function. It is supposed that forces act in the fibres only when the fibres are in tension.

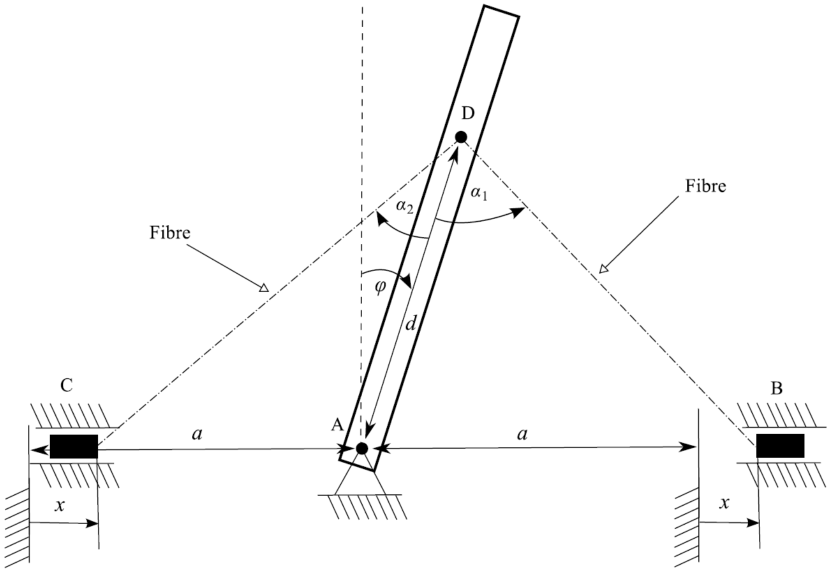

Fig. 1. Inverted pendulum actuated by the fibres.

Original length l_{v0} of the fibres is supposed to be constant and actual lengths l_{v1} and l_{v2} of the fibres should be calculated in each time

$$\begin{aligned} l_{v1} &= \sqrt{\left(d\cos\varphi\right)^2 + \left(a + x(t) - d\sin\varphi\right)^2}\ , \\ l_{v2} &= \sqrt{\left(d\cos\varphi\right)^2 + \left(a - x(t) + d\sin\varphi\right)^2}\ . \end{aligned} \tag{3}$$

The kinematic excitation is given by function

$$x(t) = x_0 \sin\left(2\pi f t\right), \tag{4}$$

where x_0 is the chosen amplitude of motion, f is the excitation frequency and t is time. The influence of the excitation frequency on the pendulum motion is investigated. Excitation in points designated B and C (see Fig. 1) is considered to be symmetrical (without any mutual phase shift) and of the same amplitude x_0.

The “light” fibre made of thin carbon fibres at Department of Mechanics, Biomechanics and Mechatronics, Faculty of Mechanical Engineering, Czech Technical University in Prague (the mass of the one carbon fibre is 3.846 grams) is considered (in contrast to [4,5,6,17], where the wattled steel wire was considered) – see Fig. 2. This fibre was chosen for the check of not exceeding the permissible load of fibres during their acting under dynamic loading due to the fact that it is supposed to be used for performing the planned experiment and its stiffness characteristics are well known (they were measured by a tensile testing machine at Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia [8]). Some of the most important model parameters (see Fig. 1) are the same as in [4,5,6,17]: $l = 1$ m, $a = 1.2$ m, $d = 0.75$ m, $I_A = 3.288$ kg·m^2 and $m = 9.864$ kg. Stiffness $k_v = 120.543 \cdot 10^3$ N/m and damping coefficient $b_v = 5 \cdot 10^{-4} \cdot k_v$ N·s/m differ from [4,5,6,17]. Stiffness k_v is calculated on the basis of [8], where it is called “approximation min – max”.

The natural frequency of the linearized system of the inverted pendulum in equilibrium position is 19.4 Hz.

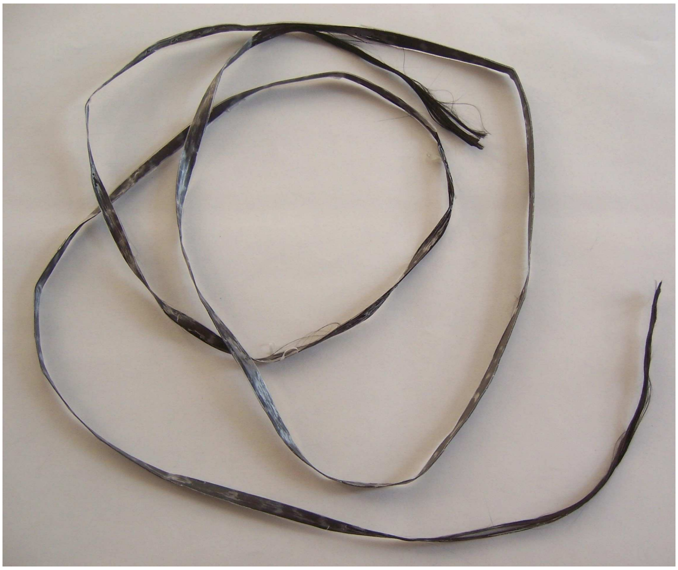

Fig. 2. “Light” fibre made of thin carbon fibres.

4. Simulations results

The kinematic excitation amplitude (defined by Eq. (4)) x_0 = 0.02 m was chosen (as in [4-7]). Excitation frequency f was considered in the range from 0.1 Hz to 200 Hz.

Time histories and extreme values of the force in the fibres and of pendulum angle φ (maximum value of static angular displacement of pendulum is φ = 1.52°; minimum value of static angular displacement of pendulum is logically φ = –1.52°) are the monitored quantities. Simulation time is 3 seconds (it differs from [4,5,6,7,16,17], where the simulation time was 10 seconds). It was tested that after this period the character of the system response to the kinematic excitation does not change. Selected results of the numerical simulations are presented in Figs 3 to 8.

Extreme values of pendulum angle φ in dependence on excitation frequencies are given in Fig. 3, extreme values of forces acting in fibres in dependence on excitation frequencies are given in Fig. 4. Selected time histories of pendulum angle and corresponding force acting in (right) fibre obtained at various parameters of system of inverted pendulum model are given in Figs 5 to 8.

From the results obtained (see Figs 3 to 8) it is evident that the pendulum motion is influenced by the excitation frequency (it is known from [4,5,6,7,16,17] already).

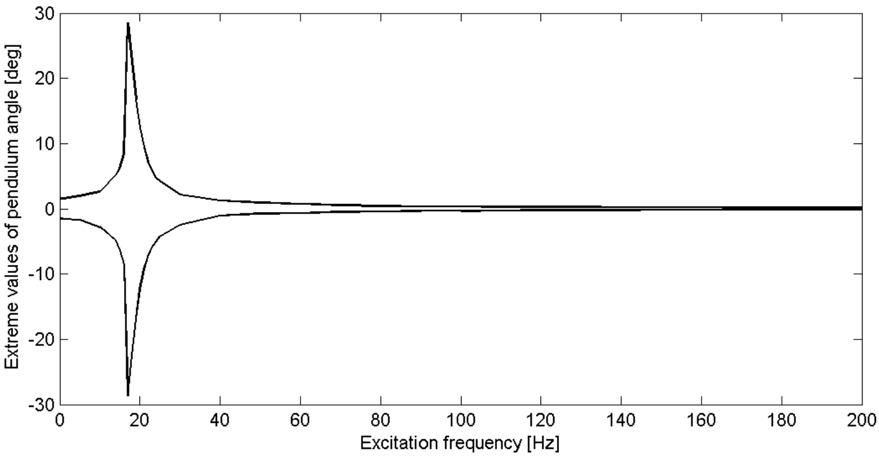

Fig. 3. Extreme values of time histories of pendulum angle φ in dependence on excitation frequencies.

As to the character of the course of forces acting in fibres the expected results were verified (see Fig 4). The greatest forces act in fibres (maximum force 34 768 N) at maximum values of pendulum angle φ, i.e. at excitation frequencies in the neighbourhood of the pendulum natural frequency 19.4 Hz. The same is valid for spring components of these forces. Naturally, damping components of forces acting in fibres are the highest (besides at excitation frequencies in the neighbourhood of the pendulum natural frequency) at the highest excitation frequencies.

The maximum quasi-static force acting in the fibre before its breaking, which was determined by tensile testing machine in [8] (sample B), is 1 069 N.

From the obtained results it is evident that for the considered conditions of kinematic excitation of fibres (x_0 = 0.02 m), the “light” carbon fibre is not suitable. Dynamic forces acting in the fibres considerably exceed maximum quasi-static forces in the fibre determined by tensile testing machine. It is necessary to use other fibres (it will be investigated) or change conditions of kinematic excitation of fibres (the suitable calculated kinematic excitation amplitude for “light” carbon fibres x_0 = 0.002 m; in addition, upper limit of the excitation frequencies is too high for the practical use in manipulators) or change some parameters of the system of the inverted pendulum (e.g. the moment of inertia of the pendulum).

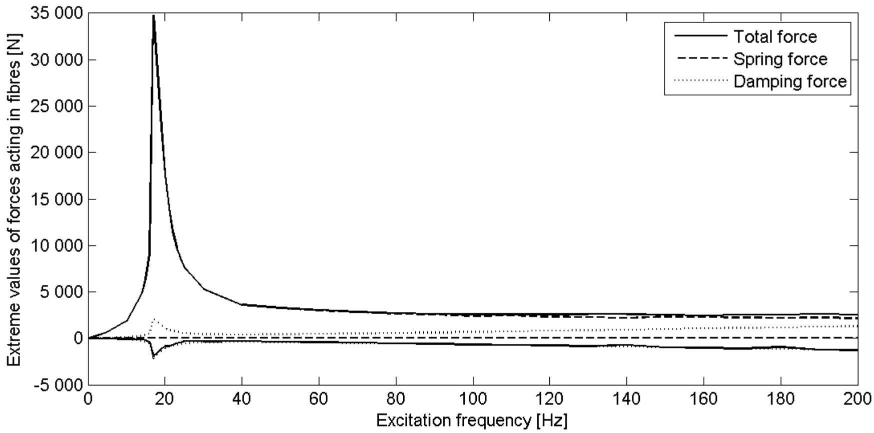

Fig. 4. Extreme values of time histories of forces acting in fibres in dependence on excitation frequencies.

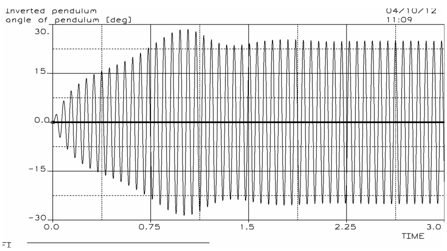

Fig. 5. Time history of pendulum angle φ [deg], excitation frequency f = 17 Hz.

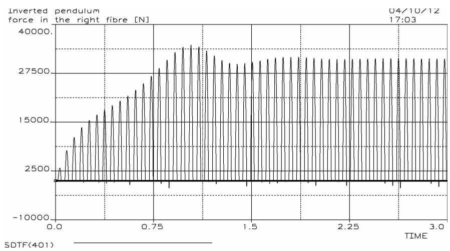

Fig. 6. Time history of force [N] acting in fibre 1, excitation frequency f = 17 Hz.

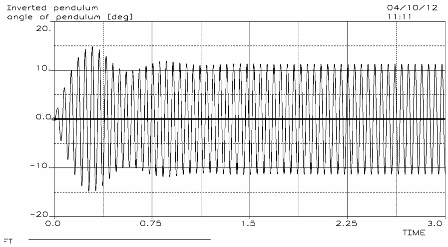

Fig. 7. Time history of pendulum angle φ [deg], excitation frequency f = 19.4 Hz.

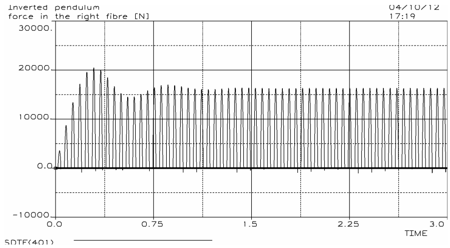

Fig. 8. Time history of force [N] acting in fibre 1, excitation frequency f = 19.4 Hz.

5. Conclusion

The check of not exceeding the permissible load of fibres during their acting under dynamic loading is given. From the obtained results it is evident that for the considered conditions of kinematic excitation of fibres (x_0 = 0.02 m), the “light” carbon fibre is not suitable. Dynamic forces acting in the fibres considerably exceed maximum quasi-static forces in the fibres determined by tensile testing machine in [8]. It is necessary to use other fibres or change conditions of kinematic excitation of fibres or change some parameters of the system of the inverted pendulum.

Experimental verification of the cable dynamics within the manipulator systems and research aimed at measuring the material properties of further selected fibres are considered important steps in further research.

Acknowledgement

The paper has originated in the framework of solving the No. P101/11/1627 project of the Czech Science Foundation entitled “Tilting Mechanisms Based on Fiber Parallel Kinematical Structure with Antibacklash Control” and institutional support for the long-time conception development of the research institution provided by Ministry of Industry and Trade of the Czech Republic.

References

[1] Chan E.H.M., *Design and Implementation of a High-Speed Cable-Based Parallel Manipulator, PhD Thesis* (University of Waterloo, Waterloo, 2005).

[2] Smrž M. and Valášek M., “New Cable Manipulators,” in *Proceeding of National Conference with International Participation Engineering Mechanics 2009*, Svratka, 2009 (Institute of Theoretical and Applied Mechanics Academy of Sciences of the Czech Republic, Prague, 2009), CD-ROM (pp. 1209-1216). ISBN 80-86246-35-2.

[3] Valášek M. and Karásek M., “HexaSphere with Cable Actuation,” in *Recent Advances in Mechatronics: 2008-2009*, Brezina T., Jablonski R., eds (Springer-Verlag, New York, 2009), pp. 239-244. ISBN 978-3-642-05021-3.

[4] Polach P. and Hajžman M., “Investigation of dynamic behaviour of inverted pendulum attached using of fibres,” in *Proceedings of 11th Conference on Dynamical Systems – Theory and Applications, Nonlinear Dynamics and Control*, Łódź, 2011 (Department of Automatics and Biomechanics, Technical University of Łódź, Łódź, 2011), pp. 403-408. ISBN 978-83-7283-448-5.

[5] Polach P. and Hajžman M., “Investigation of dynamic behaviour of inverted pendulum attached using fibres at non-symmetric harmonic excitation,” in *Book of Abstracts of EUROMECH Colloquium 524 Multibody system modelling, control and simulation for engineering design*, Enschede, 2012 (University of Twente, Enschede, 2012), pp. 42-43. ISBN 978-90-365-3338-6.

[6] Polach P. and Hajžman M., “Effect of Fibre Preload on the Dynamics of an Inverted Pendulum Driven by Fibres,” in *Proceedings of The 2nd Joint International Conference on Multibody System Dynamics*, Stuttgart, 2012 (University of Stuttgart, Stuttgart, 2012), CD-ROM.

[7] Polach P., Hajžman M., Šika Z., Mrštík J. and Svatoš P., “Effects of fibre mass on the dynamics of an inverted pendulum driven by cables,” in *Proceedings of National Colloquium with International Participation Dynamics of Machines 2012*, Prague, 2012 (Institute of Thermomechanics Academy of Sciences of the Czech Republic, Prague 2012), pp. 127-134. ISBN 978-80-87012-39-0.

[8] Zemčík R., *Determination of stiffness and strength of carbon fibres in axial direction, Research report to the order no. 11-02200707* (University of West Bohemia, Plzeň, 2011). (in Czech)

[9] Hajžman M. and Polach P., “Modelling of Cables for Application in Cable-Based Manipulators Design,” in *Proceedings of ECCOMAS Thematic Conference Multibody Dynamics 2011*, Brussels, 2011 (Université catholique de Louvain, Brussels, 2011), CD-ROM.

[10] Shabana A.A., “Flexible Multibody Dynamics: Review of Past and Recent Developments,” *Multibody System Dynamics*, **1**(2), pp. 189-222 (1997). ISSN 1384-5640.

[11] Diao X. and Ma O., “Vibration analysis of cable-driven parallel manipulators,” *Multibody System Dynamics*, **21**(4), pp. 347-360 (2009). ISSN 1384-5640.

[12] Kamman J.W. and Huston R.L., “Multibody Dynamics Modeling of Variable Length Cable Systems,” *Multibody System Dynamics*, **5**(3), pp. 211-221 (2001). ISSN 1384-5640.

[13] Wittbrodt E., Adamiec-Wójcik I. and Wojciech S., *Dynamics of Flexible Multibody Systems, Rigid Finite Element Method* (Springer, Berlin, 2006). ISBN 978-3540323518.

[14] Freire A. and Negrão J., “Nonlinear Dynamics of Highly Flexible Partially Collapsed Structures,” in *Proceedings of III European Conference on Computational Mechanics, Solids, Structures and Coupled Problems in Engineering*, Lisbon, 2006, (Springer, Dordrecht, 2006), CD-ROM. ISBN 978-1-2040-4994-1.

[15] Wei M.H., Xiao Y.Q. and Liu H.T., “Bifurcation and chaos of a cable-beam coupled system under simultaneous internal and external resonances,” *Nonlinear Dynamics*, **67**(3), pp. 1969-1984 (2012). ISSN 0924-090X.

[16] Polach P., Hajžman M. and Tuček O., “Validation of the point-mass modelling approach for fibres in the inverted pendulum model,” in *Proceedings of National Conference with International Participation Engineering Mechanics 2012*, Svratka, 2012 (Institute of Theoretical and Applied Mechanics Academy of Sciences of the Czech Republic, Prague, 2012), CD-ROM.

[17] Polach P. and Hajžman M., “Approaches to the Modelling of Inverted Pendulum Attached Using of Fibres,” in *Proceedings of 4th International Conference on Modelling of Mechanical and Mechatronic Systems 2011*, Herľany, 2011, (Technical University of Košice, Košice, 2011), CD-ROM (pp. 408-416). ISBN 978-80-553-0731-2.