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Recursive Identification Algorithms Library in SIMULINK

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Abstract. The article describes a simple library for recursive parameter estimation of linear dynamic models ARX, ARMAX and OE in SIMULINK development environment. Several recursive identification methods were implemented in this library: Least Squares Method (RLS), Recursive Leaky Incremental Estimation (RLIE), Damped Least Squares (DLS), Adaptive Control with Selective Memory (ACSM), Instrumental Variable Method (RIV), Extended Least Squares Method (RELS), Prediction Error Method (RPEM) and Extended Instrumental Variable Method (ERIV). To cope with tracking the time-variant parameters several forgetting factor and modification of basic algorithm are taken into consideration.

Introduction

There exist many complex packages for system identification purposes in MATLAB and SIMULINK environment. These toolboxes provide solution to wide range of the problems from the area of system identification, e.g. System Identification Toolbox [1] and Continuous Identification Toolbox [2]. There also exist many special-purpose programs and libraries for MATLAB and SIMULINK, e.g. Idtool [3]. These simple tools provide solution to specific problems from the concrete part of the area of system identification.

The proposed Recursive Identification Algorithms Library (RIA) fall into category of simple libraries for SIMULINK environment and is designed for recursive estimation of the parameters of the linear dynamic models ARX, ARMAX and OE. The Recursive Identification Algorithms Library consists of several user-defined blocks. These blocks implement several recursive identification algorithms: Least Squares Method (RLS) and its modifications, Recursive Leaky Incremental Estimation (RLIE), Damped Least Squares (DLS), Adaptive Control with Selective Memory (ACSM), Instrumental Variable Method (RIV), Extended Least Squares Method (RELS), Prediction Error Method (RPEM) and Extended Instrumental Variable Method (ERIV). The Recursive Identification Algorithms Library can be used for simulation or real-time experiment (e.g. Real Time Toolbox) in educational process when it is possible to demonstrate the properties and behaviour of the recursive identification algorithms and forgetting factors under various conditions.

Model structure

The basic step in identification procedure is the choice of suitable type of the model. General linear model takes the following form:

$$y(k)=\frac{B(q^{-1})}{A(q^{-1})F(q^{-1})}u(k)+\frac{C(q^{-1})}{A(q^{-1})D(q^{-1})}n(k) \qquad (1)$$

where

$$
\begin{aligned}
&A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \\
&B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{nb} q^{-nb} \\
&C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc} \\
&D(q^{-1}) = 1 + d_1 q^{-1} + \ldots d_{nd} q^{-nd} \\
&F(q^{-1}) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}
\end{aligned}
\tag{2}
$$

are shift operators polynomials and $y(k)$, $u(k)$ are output and input signals. White noise $n(k)$ is assumed to have zero mean value and constant variance.

All linear models can be derived from general linear model by simplification. In the Recursive Identification Library following linear dynamic models are taken into consideration. These are ARX, ARMAX, OE models.

ARX model (C=D=F=1):

$$
y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + \frac{1}{A(q^{-1})} n(k) \tag{3}
$$

ARMAX model (D=F=1):

$$
y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + \frac{C(q^{-1})}{A(q^{-1})} n(k) \tag{4}
$$

OE model (A=C=D=1):

$$
y(k) = \frac{B(q^{-1})}{F(q^{-1})} u(k) + n(k) \tag{5}
$$

Recursive parameter estimation

The recursive parameter estimation algorithms are based on the data analysis of the input and output signals from the process to be identified. Many recursive identification algorithms were proposed [4, 5]. In this part several recursive algorithms with forgetting factors implemented in Recursive Identification Algorithms Library are briefly summarized.

Recursive instrumental variable (RIV). It can be shown that if the process does not meet the noise assumption made by the ARX model, the parameters are estimated biased and non-consistent. This problem can be avoided using instrumental variable method. Choice of instrumental variable determines behaviour of the IV method in usage. Some common choices for generating instruments are proposed in [5].

Recursive leaky incremental estimation (RLIE). With any parameter estimation algorithm, it is unavoidable that certain errors or noise will be present in the estimation loop. It can be shown when estimation algorithm contains an integrator in the estimation loop it may reduce its stability margin and accumulate the error effect, causing possible parameter estimate drift. To overcome this problem, it can be possible to modify the algorithm structure so that integral action is somewhat blunted. This can be achieved by introducing some leakage into the integration. The recursive leaky incremental estimation [6] can be describes as follows (Eq. 6):

$$\hat{\mathbf{\Theta}}(k)=\gamma\hat{\mathbf{\Theta}}(k-1)+\Gamma\hat{\mathbf{\Theta}}(k)$$
$$\Gamma\hat{\mathbf{\Theta}}(k)=\Gamma\hat{\mathbf{\Theta}}(k-1)+\mathbf{C}(k)\phi(k)\left(y(k)-\gamma\hat{\mathbf{\Theta}}(k-1)-\phi^{T}(k)\Gamma\hat{\mathbf{\Theta}}(k-1)\right) \tag{6}$$
$$\mathbf{C}(k)=\frac{1}{\lambda}\left(\mathbf{C}(k-1)-\frac{\mathbf{C}(k-1)\phi(k)\phi^{T}(k)\mathbf{C}(k-1)}{\lambda+\phi^{T}(k)\mathbf{C}(k-1)\phi(k)}\right)$$

Where Γ denotes the stabilizing operator, defined as $\Gamma=1-\gamma q^{-1}$ and $\gamma\in[0,1]$ is the stabilizing parameters which is preselected by the user.

Damped least squares algorithm (DLS). Damped least squares (DLS) algorithm is an extended version of the recursive simple least squares (RLS) algorithm [7]. The DSL algorithm is more appropriate for adaptive control, since it weights increments of estimated parameter vector (Eq. 7). This gives more control on the adaptation rate.

The DLS criterion is

$$J\left(\hat{\mathbf{\Theta}}\right)=\sum_{k=t-N}^{t}\lambda^{t-k}\left[y(k)-\phi^{T}(k)\hat{\mathbf{\Theta}}(k)\right]^{2}+\left[\mathbf{\Lambda}_{d}(k)\left(\hat{\mathbf{\Theta}}(k)-\hat{\mathbf{\Theta}}(k-1)\right)\right]^{2} \tag{7}$$

The weighting matrix $\mathbf{\Lambda}_{d}(k)$ is diagonal and weights the parameters variations (Eq. 8). For an n-parameters model,

$$\mathbf{\Lambda}_{d}(k-1)=diag\left[\alpha_{1}(k)\ \alpha_{2}(k)\ldots\alpha_{n}(k)\right] \tag{8}$$

A standard form of the DLS algorithm is given

$$\hat{\mathbf{\Theta}}(k)=\hat{\mathbf{\Theta}}(k-1)+\mathbf{L}\left[y(k)-\hat{\mathbf{\Theta}}^{T}(k)\phi(k)\right]+\mathbf{C}(k)\lambda(k)\mathbf{\Lambda}_{d}(k)\left[\hat{\mathbf{\Theta}}(k-1)-\hat{\mathbf{\Theta}}(k-2)\right]$$
$$\mathbf{L}(k)=\frac{\mathbf{C}(k-1)\phi(k)}{\lambda(k)+\phi^{T}(k)\mathbf{C}(k-1)\phi(k)}$$
$$\mathbf{C}(k)=\frac{1}{\lambda(k)}\left(\mathbf{C}'(k-1)-\frac{\mathbf{C}'(k-1)\phi(k)\phi^{T}(k)\mathbf{C}'(k-1)}{\lambda+\phi^{T}(k)\mathbf{C}'(k-1)\phi(k)}\right)$$
$$\mathbf{C}'(k)=\mathbf{C}'(k-1)-\sum_{i=1}^{n}\frac{\mathbf{C}'_{i-1}(k-1)r_{i}r_{i}^{T}\mathbf{C}'_{i-1}(k-1)\alpha'_{i}(k)}{1+r_{i}^{T}\mathbf{C}'_{i-1}(k-1)r_{i}\alpha'_{i}(k)} \tag{9}$$
$$\mathbf{C}'_{1}(k-1)=\mathbf{C}'(k-1)-\frac{\mathbf{C}'_{i-1}(k-1)r_{i}r_{i}^{T}\mathbf{C}'_{i-1}(k-1)\alpha'_{i}(k)}{1+r_{i}^{T}\mathbf{C}'_{i-1}(k-1)r_{i}\alpha'_{i}(k)}$$
$$\mathbf{C}'_{0}(k-1)=\mathbf{C}(k-1)$$
$$\alpha'_{i}(k)=\frac{\alpha_{i}(k)-\lambda(k)\alpha_{i}(k-1)}{\lambda(k)}$$

Recursive pseudolinear regression (RPLR). Recursive pseudolinear regression method (RPLR) is used for parameter estimations of ARMAX and OE model. Formally it takes the same form as RLS [4, 5]. However, the regression and parameter vectors are different. The RPLR converges much more slowly than RLS.

Adaptive control with selective memory (ACSM). Adaptive control with selective memory [8] updates parameter estimates only when there is new information present. The information increases and estimator eventually stops. The parameter estimates are updated only when the information matrix and/or estimated variance of the prediction error increases.

The algorithm consists of several steps, equations (Eq. 10) - (Eq. 13):

Step 0: Choose $r_0 > 0, \mathbf{\Theta}(0), \mathbf{C}(0) > 0, 1 < M_0 < \infty$

$$Set\ r(0) = r_0 > 0, \sigma = 1 - \frac{1}{M_0}, \varepsilon_0 = \frac{1}{M_0}.$$

Step 1:

$$r(k) = \max\left\{\sigma r(k-1) + (1-\sigma)\hat{e}(k-1)^2, r_0\right\} \tag{10}$$

Step 2: $Set\ B(k) = 0$ and

$$A(k) = \begin{cases} 1 & if \dfrac{\phi^T(k)\mathbf{C}(k-1)\phi(k)}{r(k)} \geq \varepsilon_0 \\ 0 & otherwise \end{cases} \tag{11}$$

Step 3: $If\ A(k) = 0$, *set*

$$B(k) = \begin{cases} 1 & if\ r(k) \geq \max_{1 \leq i \leq k} r(k-i) \\ 0 & otherwise \end{cases} \tag{12}$$

$$Set\ \Delta(k) = A(k) + B(k)$$

Step 4:

$$\begin{aligned} \hat{\mathbf{\Theta}}(k) &= \hat{\mathbf{\Theta}}(k-1) + \Delta(k) \frac{\mathbf{C}(k-1)\phi(k)}{r(k) + \phi^T(k)\mathbf{C}(k-1)\phi(k)} \hat{e}(k) \\ \mathbf{C}(k) &= \mathbf{C}(k-1) - \Delta(k) \frac{\mathbf{C}(k-1)\phi(k)\phi^T(k)\mathbf{C}(k-1)}{r(k) + \phi^T(k)\mathbf{C}(k-1)\phi(k)} \end{aligned} \tag{13}$$

Step 5: Set $k = k + 1$ and go to step 1

Extended recursive instrumental variable method (ERIV). This method ensures improved accuracy and greater speed of convergence than RIV. The method is based on choice of instruments vector which has more elements than there are parameters in the model to be estimated. Derivation of this algorithm can be found in [5]. Instruments can be chosen according to [5].

Recursive prediction error method (RPEM) allows the online identification of all linear model structure. Since all model structure except ARX are nonlinearly parameterized, no exact recursive algorithm can exist; rather some approximations must be made [5]. In fact, the RPEM can be seen as a nonlinear least squares Gauss-Newton method.

Recursive identification algorithms library (RIA)

The Recursive Identification Algorithm Library is designed for recursive parameter estimation of linear dynamic model ARX, ARMAX, OE using recursive identification methods: Least Squares Method (RLS), Recursive Leaky Incremental Estimation (RLIE), Damped Least Squares (DLS), Adaptive Control with Selective Memory (ACSM), Instrumental Variable Method (RIV), Extended Least Squares Method (RELS), Prediction Error Method (RPEM) and Extended Instrumental Variable Method (ERIV). The Recursive Identification Algorithm Library is depicted in Fig. 1. The Library consists of 18 user-defined blocks and is designed for MATLAB&SIMULINK environment. Each block is realized as an s-function.

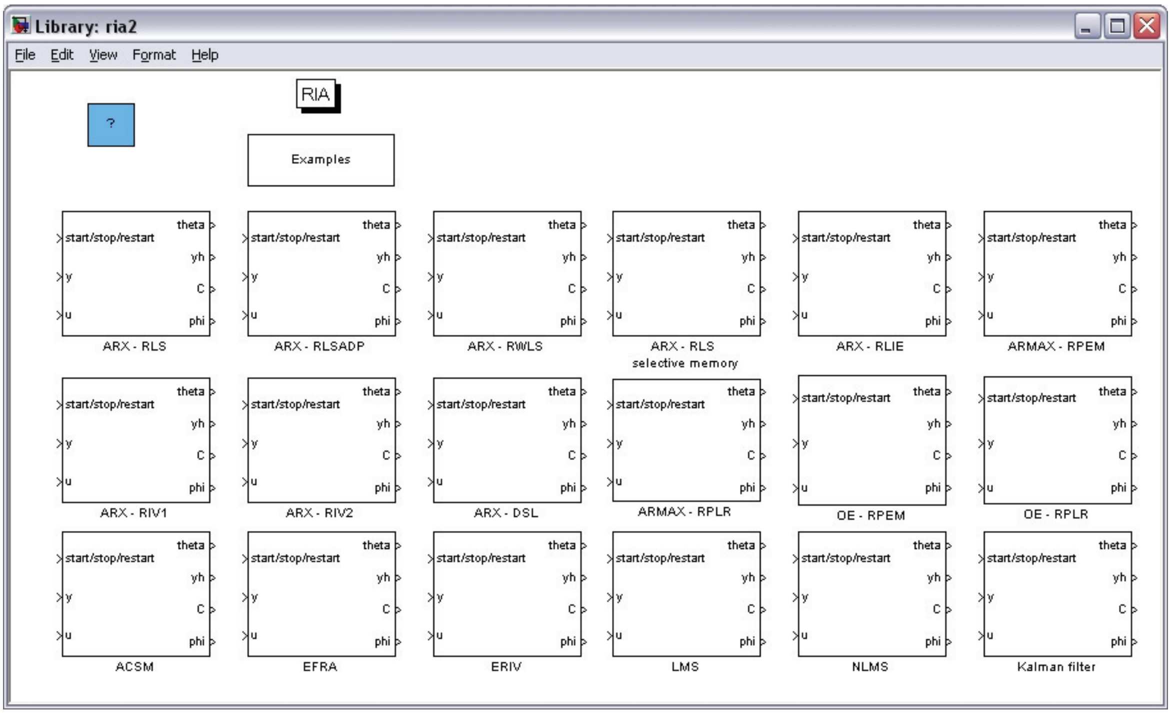

Fig. 1 - Recursive Identification Algorithms Library

Each block is masked by user-defined dialog. Several necessary input parameters should be input through this dialog. These are: type of forgetting factor and its value, degrees of polynomials, sampling period, initial values of parameter estimate, covariance matrix and data vector, etc. Each block also contains the help describes the meaning of each parameter, inputs and outputs and used recursive identification algorithms. Example of input dialog is shown in Fig. 2.

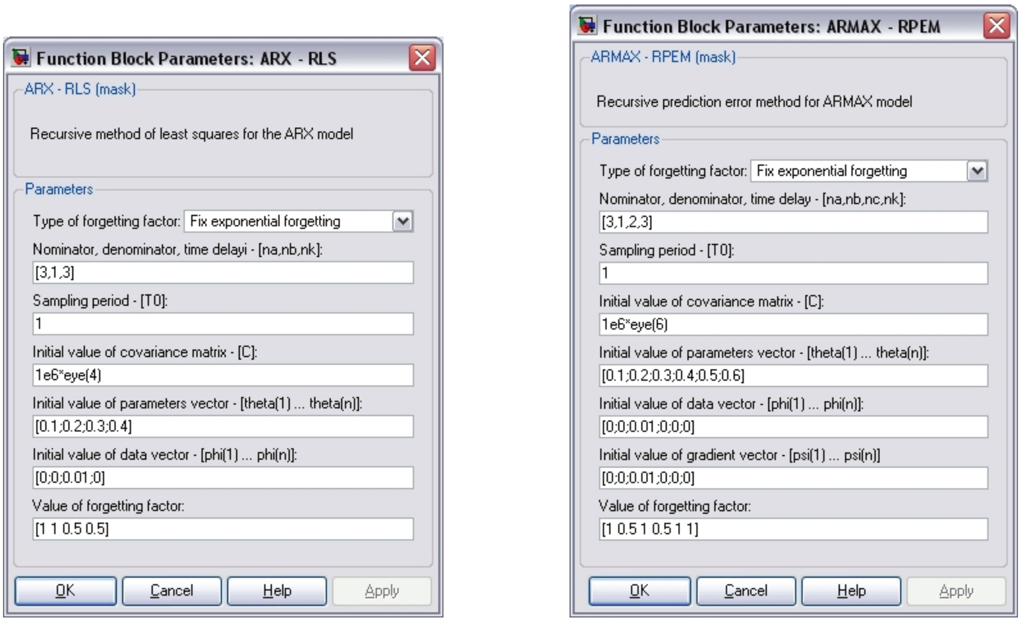

Fig. 2 – Example of input dialog of the identification blocks

Input/output data from object under identification process are inputs to the identification block. Another input (start/stop/restart) is used for control the identification algorithm. This input provides possibility of start, stop and restart the identification algorithm in selected instant of time. Outputs of the block are estimate of parameter vector, one-step prediction of output of model, covariance matrix and data vector. Example of application of the identification block in the model is illustrated in Fig. 3.

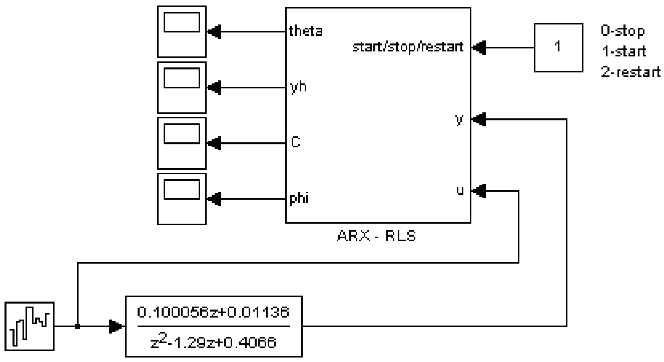

Fig. 3 - Example of application of identification block

Conclusion

The Recursive Identification Algorithm Library is designed for recursive parameter estimation of linear dynamic model ARX, ARMAX, OE using recursive identification methods. The library can be used e.g. in identification part of self-tuning controller or in educational process when it is possible to demonstrate the properties and behaviour of the recursive identification algorithms and forgetting factors under various conditions. Proposed library can be used not only in educational process to demonstrate the behavior and properties of recursive identification algorithms, but for example in connection with such Real Time Toolbox provide the possibility to real time estimation of the parameters of the model of real systems.

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