Determination of Initial Shear Modulus of Woven Fabrics in Various Directions

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Abstract: Shear behavior is one of the most important mechanical characteristics that contribute to performance and appearance of woven fabrics. Because of the anisotropy, shear properties of woven fabric are tested in various directions. This paper describes an experimental study of shear properties of woven fabric when shear force acts on specimens that are cut at different angles to the direction of the weft. Tests are conducted on woven fabric specimens that are fastened in two parallel clamps that are placed in tensile tester. Orthogonal plain weave cotton woven fabric was used. Based on diagram of the measured values of force and corresponding vertical displacement, shear angles and corresponding shear stresses are calculated. Initial shear modulus of woven fabrics was determined experimentally and theoretically in various directions.

Keywords: woven fabric; initial shear modulus; pure shear; anisotropy; shear angle.

1 Introduction

During shear deformation, the woven fabric yarns experience large angular variation between warp and weft yarns. Test methods shear properties of woven fabrics are listed in the existing literature [1]. To understand the mechanisms of woven fabric shear behavior, shear apparatus which measure woven fabric shear properties are described [2]. At the beginning, researches have mainly focused on the shear behavior of woven fabrics in both principal directions because it affects much woven fabric behavior, and later their attention has been paid to woven fabric shear properties in various directions. Anisotropy is the characteristics of woven fabric that affects its physical and mechanical properties [3]. Determination of shear stresses and strains in various directions involves complex mechanisms that provide information about shear properties of woven fabrics in various directions where the angles between two sets of yarns change in the intersecting points. The aim of this study is to determine the shear angle, shear forces of woven fabric using clamps that are specially designed in laboratory for measuring shear forces. Shear force is acting on woven fabric specimens that are cut at different angles to the weft direction. The influence of anisotropy to the initial shear modulus values of a woven fabric was analyzed. The degree of agreement between experimental results and calculated obtained values of the initial shear modulus was determined. The structure of the apparatus and the measurement procedures are illustrated in this paper. Woven fabric specimens that are fastened in two parallel clamps that are placed in tensile tester will be used for experimental determination of shear properties of the woven fabric.

2 Theoretical Overview

The woven fabrics are elastic orthotropic materials at very small deformation that are defined as orthotropic plates with two mutually perpendicular planes of elastic symmetry [4]. The x-axis is in the weft direction, and the y-axis is the warp direction of woven fabric. For a particular state of plane stress in element we can determine the plane in which only shear stresses act and normal stresses are zero. Such a case of stress is called a pure shear. Isolated element of woven fabric *ABCD* which is subjected to pure shear is shown in Fig. 1. During shear deformation, the woven fabric yarns experience large angular variation between warp and weft yarns.

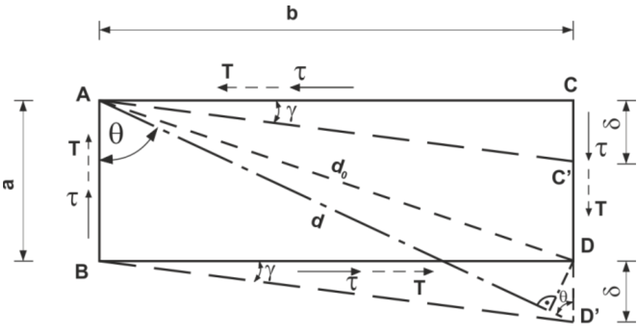


Fig. 1: Woven fabric element subjected to pure shear - rectangular shape of element.

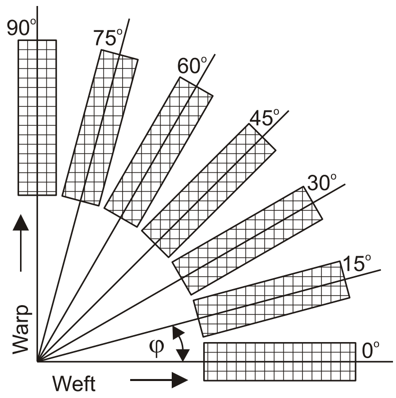


Fig. 2: Schematic view of directions (angles) of cutting specimens.

If the side AB is motionless, then under the action of shear force T which acting in the plane of the element side shear stress τ will occur and it will come to the appearance of shearing the side DC parallel with the side AB for the amount CC'=DD'=δ (mm) which is called the absolute shear (Fig. 1). Change the right angle is denoted with γ (rad) and is called the relative shear strain or shear angle. It is a measure of deformation. During the elastic deformation the shear angle γ is very small and it is equal to the absolute shear divided by the spacing between the shear plane, the Eq. (1):

$$tg\gamma \cong \gamma = \frac{\delta}{b} \qquad (1)$$

Shear stress τ (N/mm^2 = MPa) of woven fabric can be directly calculated using shear forces. It is assumed that the thickness of the woven fabric t (mm) is constant during shearing. Then the shear stress is calculated using the Eq. (2)

$$\tau = \frac{T}{A} \qquad (2)$$

where are: T (N) is a shear force acting on the side DC of woven fabric element as a result of stress (assuming a uniform distribution of shear stresses on the surface side), A (mm^2) is a surface side DC of woven fabric element.

2.1 Shear modulus of woven fabrics

The functional relationship between stress and strain cannot be determined theoretically, but only by experimental testing of samples made of certain materials. Mechanical properties are mainly investigated within the area of elasticity which means in terms of low load [5]. It is assumed that force- shear angle curve for woven fabric is approximate straight line before yield point. Therefore, elastic performance equation can apply here. The shear modulus is the initial, linear elastic slope of the stress-strain curve in shear. The connection between the shear angle γ and the shear stress τ is shown by Eq. (3) which represents the Hooke's law of shear and applies for elastic, homogeneous isotropic material in a linear region, i.e. where the relationship between shear stress and shear strain is linear. G (N/mm^2 = MPa) is shear modulus.

$$\tau = G \cdot \gamma \qquad (3)$$

For orthotropic elastic materials, shear modulus G_{kl}, in various directions of cutting samples, i.e. in the coordinate system k, l whose axes do not coincide with the main axes x, y is obtained by using the expression for the transformation of the elastic constants, which states [5]:

$$\frac{1}{G_{kl}} = \left(\frac{1}{E_x} + \frac{1}{E_y} + \frac{2 \cdot \nu_{xy}}{E_x}\right) \cdot 4\cos^2\varphi \cdot \sin^2\varphi + \frac{1}{G_{xy}} \cdot \left(\cos^2\varphi - \sin^2\varphi\right)^2 = \frac{1}{G_\varphi} \qquad (4)$$

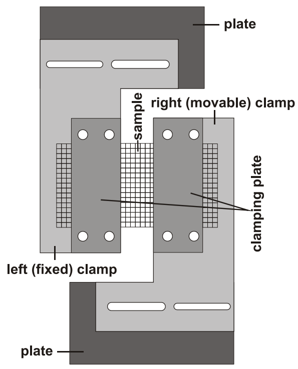
φ is cutting angle of the samples due to the weft (Fig. 2), E_x, E_y are modulus of elasticity in two main directions (weft direction φ=0°, warp direction φ=90°); G_{xy} is shear modulus between both principal directions; v_{xy} is the Poisson's ratio. Shear modulus G_{kl} changes depending on the angle φ, and will be labeled as $G_\varphi=G_{kl}$. The numerical values of E_x, E_y, G_{xy}, v_{xy} are obtained by experimental testing of woven fabric samples in the laboratory. The theoretical treatment suggests that measurements of modulus in two directions (weft and warp direction) are insufficient to define a woven fabric's shear modulus. An investigation of the third direction is therefore necessary, and the most convenient direction is φ=45°. So, the measurements were carried out in three directions by considering samples cut along the warp, weft, and 45° directions. Shear modulus G_φ in any given direction can be predicted from the Eq. (5) when its values in the warp or weft directions and under the angle of 45° are measured:

$$\frac{1}{G_\varphi}=\left(\frac{4}{G_{45^0}}\right)\cos^2\varphi\sin^2\varphi+\frac{1}{G}\left(\cos^2\varphi-\sin^2\varphi\right)^2 \quad (5)$$

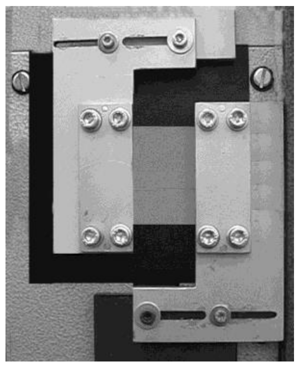
If differences in the shear modulus values between the warp and weft directions are large, we take the average value in both principal directions to calculate the shear modulus in various directions.

3 Experimental Part

The experimental study of shear properties of the woven fabric when the shear force acts on the samples that are cut at different angles 0°, 15°, 30°, 45°, 60°, 75°, 90° to the direction of the weft was conducted (Fig. 2). The values of the shear force in relation with shear angle were measured. Based on the experimentally obtained values, initial shear modulus was calculated in various directions. To carry out this study, orthogonal plain weave cotton woven fabric was available. Its fabric code is S23 and structural properties are: weft and warp yarn count is 30 tex, warp yarn density is 23 cm^{-1}, weft yarn density is 23 cm^{-1}, mass is 150 g/m^2, thickness t=0.41 mm. Before testing all samples were conditioned under standard atmospheric conditions (relative humidity 65 ± 2%, temperature of 20 ± 2 ° C). For each cutting direction, the average values obtained from five measurements will be shown in diagrams and will be used to calculate the initial shear modulus. For determination the shear properties of the fabric, clamps for shearing woven fabrics are designed, manufactured and schematically shown in Fig. 3a, and consist of left (fixed) clamp and right (movable) clamp. The force acting on the right clamp causes its vertical displacement. Left clamp is placed on the upper plate on which there is a measuring probe, and the right clamp to the lower plate on which the movable clamp is usually placed.



a) schematic view



b) photo

Fig. 3: Sample testing in clamps.

Distance between the left and right clamp can be adjusted in the range 0-50 mm. The maximum specimen size that can be fixed within the clamp is 75 mm. For this testing, specimens were cut with dimensions 125 x 75 mm, fastened in clamps of the instrument at a distance of b=25 mm, and subjected to a force acting in the plane of a fixed side of specimen till rupture (Figure 3b). Sample dimensions are: a=75 mm and b=25 mm.

A tensile tester Statimat M German manufacturer "Textechno" was used for testing. Two parallel clamps at a distance 25 mm are fixed on tensile tester and pulling speed of right clamp is 100 mm/min.

3.1 Overview of Testing Results

Diagrams (T- δ) of average values of test results of action of force T and the vertical displacements δ until break for samples that are cut at different angles to the direction of the weft are shown in Fig. 4.

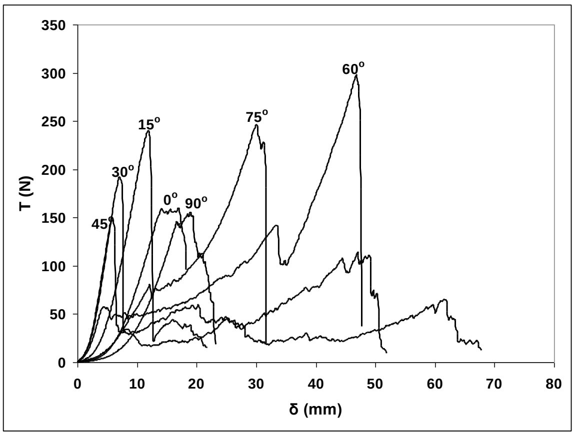


Fig. 4: Diagram T- δ (force- vertical displacement) for sample S23

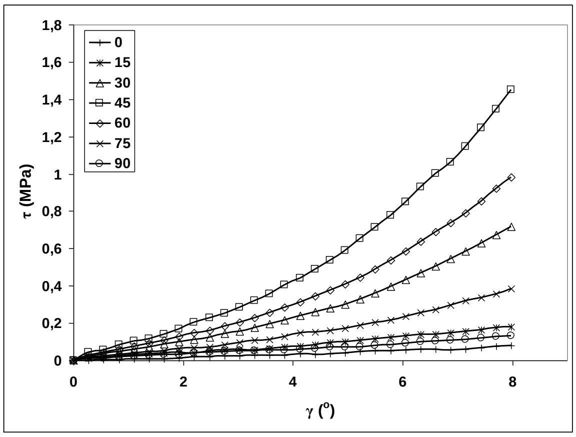


Fig. 5: Diagram τ-γ (shear stress–shear angle) for sample S23.

Based on diagrams of the measured values of force T and corresponding vertical displacement δ from Fig. 4, the average values of the shear angle γ and corresponding shear stress τ are calculated using Eq. (1) and Eq. (2). These values are shown with diagrams in Fig. 5, up to the value of the shear angle $\gamma = 8°$. During the woven fabric shearing, buckling in the woven fabric specimen occurs at a certain shear angle. According to Kawabata [6], woven fabric specimens which are subjected to the shear after exceeding the value of the shear angle $\gamma = 8°$ they tend to buckle. Shear angle $\gamma = 8°$ corresponds to vertical displacement δ=6.98 mm. The appearance of buckling causes errors in the measurement results.

In Fig. 5, at the same shear angle the highest value of shear stress τ appears in the samples which have been cut at angle of 45°. The values of shear stresses τ are very similar for complementary angles of the cutting samples. This is because balanced orthogonal plain weave fabric was considered. Shear stress τ takes on the minimum values for warp and weft directions.

3.2 Determining the initial shear modulus

Based on the experimentally obtained force-shear angles curves, the values of initial shear modulus were obtained and compared with the corresponding calculated values. Deviation in percentage between experimental and calculated values of initial shear modulus will also be calculated.

3.2.1 Experimental values of the initial shear modulus

From the presented diagrams, in Fig. 5, the values of shear force in the elastic range are used. We determined shear modulus G_φ from a particular region on force – shear angle curve that is determined by monitoring the experimental data obtained from an experimental set-up with regression control chart [7]. In this area of the curve, the relationship between shear stress and shear strain is linear. The shear modulus G_φ is defined as a slope of its shear stress- shear strain curve (τ- γ) in the elastic deformation region. The Hooke's law for shearing can be applied:

$$G_\varphi = tg\alpha = \frac{\tau}{\gamma} = \frac{T}{\gamma \cdot a \cdot t}\left(MPa\right) \tag{6}$$

where A=a·t is the area of the sample in which shear force acting.

Using values T and γ in elastic range and using Eq. (6), the average values of initial shear modulus G_φ in relation to an arbitrary direction of cutting of the woven fabric samples are calculated. Linear regression equations are placed on the shear stress- shear strain curves in the elastic range. In Fig.6, the slope of the curve, i.e. the coefficient of line direction represents the shear modulus G_φ for sample S23 at angles 0°, 45° and 90°.

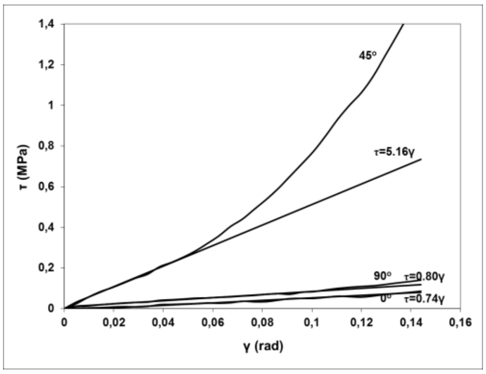


Fig. 6: Diagram of the shear stress-shear angle (τ- γ) and regression lines for woven fabric sample S23.

Tab. 1: Experimentally obtained values of shear modulus G_φ (MPa).

Sample	$G_{0°}$	$G_{15°}$	$G_{30°}$	$G_{45°}$	$G_{60°}$	$G_{75°}$	$G_{90°}$
S23	0.74	1.12	2.21	5.16	2.38	1.01	0.80

The obtained experimental values of shear modulus G_φ in dependence on the change of the cutting angle of the sample are given in Tab. 1. The diagram of experimental values G_φ for each 15° is shown in Fig. 7.

The diagram is almost symmetrical curve in relation to the angle of 45°. At that angle G_φ assumes the highest value for all woven fabric samples because initial slope of the stress-strain curve in shear is the biggest at that angle, Fig. 6. When the samples are cut in the warp direction (φ=90°) and weft direction (φ=0°) shear modulus G_φ have the lowest value because initial slope of the stress-strain curve in shear is the smallest at that angle, Fig. 6. The values G_φ in the warp and weft direction are almost equal for each woven fabric sample or it is observable that shear modulus G_φ are almost equal for complementary angles. This is because balanced orthogonal plain weave fabric was considered.

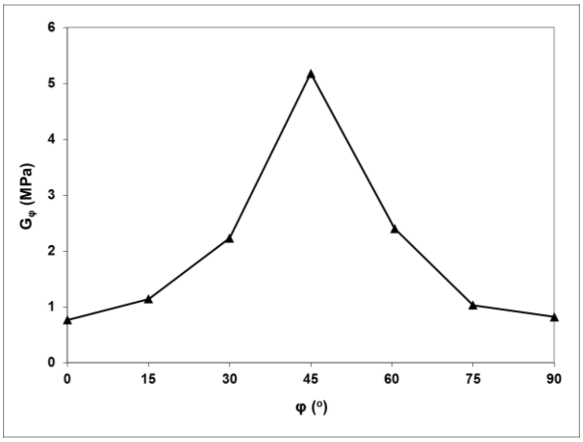


Fig. 7: Diagram of experimental values of shear modulus G_φ (MPa) for each 15° for S23.

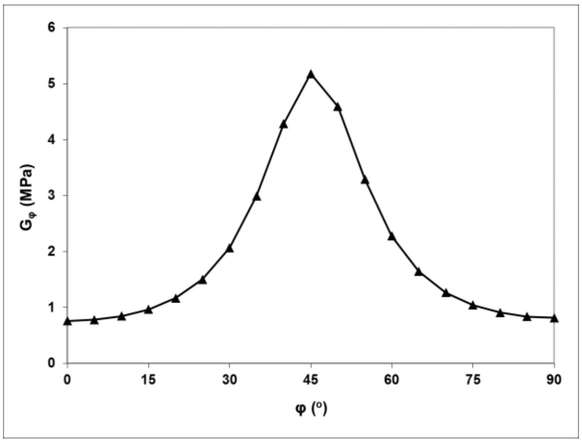


Fig. 8: Diagram of calculated values of shear modulus G_φ (MPa) for each 5° for S23.

3.2.2 Calculation of initial shear modulus in relation to an arbitrarily selected coordinate system

According to Eq. (5) and based on the experimental data $G_{0°}$ or $G_{90°}$ and $G_{45°}$ from Tab. 1, the values of initial shear modulus G_φ were calculated, depending on the change of the cutting angle of the sample. The calculated values of G_φ for each 15° are shown in Tab. 2.

Tab. 2: The theoretically calculated values of initial shear modulus G_φ (MPa).

Sample	$G_{0°}$	$G_{15°}$	$G_{30°}$	$G_{45°}$	$G_{60°}$	$G_{75°}$	$G_{90°}$
S23	0.74	0.94	2.05	5.16	2.18	1.02	0.80

The diagram of their calculated values G_φ (MPa) for each 5° is shown in Fig. 8. Tab. 3 shows deviations in percentage between experimental values G_φ from Tab. 1 and calculated values G_φ from Tab. 2. Deviations D (%) are calculated using the Eq. (7):

$$D = \frac{G_{\varphi,\exp} - G_{\varphi,calc}}{G_{\varphi,\exp}} \cdot 100(\%) \tag{7}$$

Tab. 3: Deviations D in (%) between experimental values and calculated values G_φ.

Sample	0°	15°	30°	45°	60°	75°	90°
S23	0.0	16.1	7.2	0.0	8.4	-1.0	0.0

In the warp (90°) and weft (0°) directions and under the angle of 45°, deviations in percentage between experimental and calculated values of the initial shear modulus G_φ are 0%. It follows from the Eq. (5) due to the periodicity of the sin and cos functions for these values. Negative deviation values show that the obtained calculated values G_φ are higher than the experimental values of G_φ.

4 Conclusion

Woven fabrics can be defined as orthotropic materials for which in the linear elastic range of the curve shear stress-shear angle can apply Hooke's law for anisotropic material behavior in calculating shear modulus when samples of woven fabric are cut in an arbitrary direction. The shear properties of the woven fabric are determined when the shear force acts on woven fabric specimens which are cut at different angles to the direction of the weft. Because of the anisotropy of woven fabric its shear modulus are changing in various directions. The shear modulus G_φ is almost symmetrical to the angle of 45° and the maximum value is reached exactly at that angle.

A good agreement between experimental results and the calculated obtained values of the initial shear modulus was shown, so the theoretical equations can be used with high accuracy to calculate the initial shear modulus of the woven fabric in various directions. Therefore, the measurements need to be implemented when shear force acting only on the specimens that were cut in the warp and weft direction and at an angle of 45°. The woven fabric is subjected to the shear up to $\gamma = 8°$ due to the fact that after this point woven fabric specimen tended to buckle.

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