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The Use of Geometric Mechanics Concept to Modeling of Ackerman Steered Car-like Vehicle

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Abstract. This paper provides an approach of differential geometry for modelling of the four-wheeled Ackermann-steered vehicle. Nonholonomic constraints which represent no-slip and no-slide conditions are modelled in the local coordinate systems by iterative Jacobian method. In this paper are derived kinematic equations of motion of the vehicle moving in plane.

Introduction

The motion characteristics of a vehicle play general role in planning its path. The vehicle moving in plane has generally three degrees of freedom - translation along two axes and rotation about axis perpendicular to the plane. But not every one of these movements is possible. It is necessary to consider nonholonomic constraints resulting from velocity conditions imposed on wheels. This means that when the vehicle is in motion there must be no-slip and no-slide on wheels.

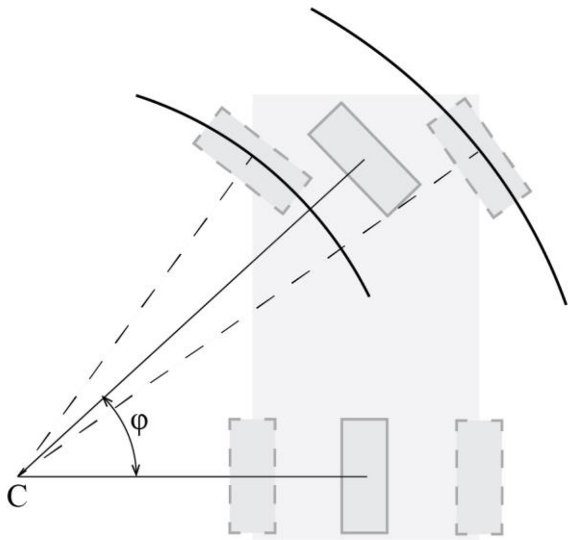

Fig. 1 Geometry of the Ackermann steered vehicle

The importance of Ackermann steering geometry (Fig. 1) is in prevention of wheels to slip sideways while vehicle is moving along a curved path. Geometrical solution is such that all wheels have their axes arranged as the radii of circles with common center.

System modelling

Based on this idea we can simplify the four wheeled vehicle model into two wheeled bicycle model as is shown in (Fig. 2). The motion characteristics of this simplified model are the same as characteristics of four wheeled vehicle.

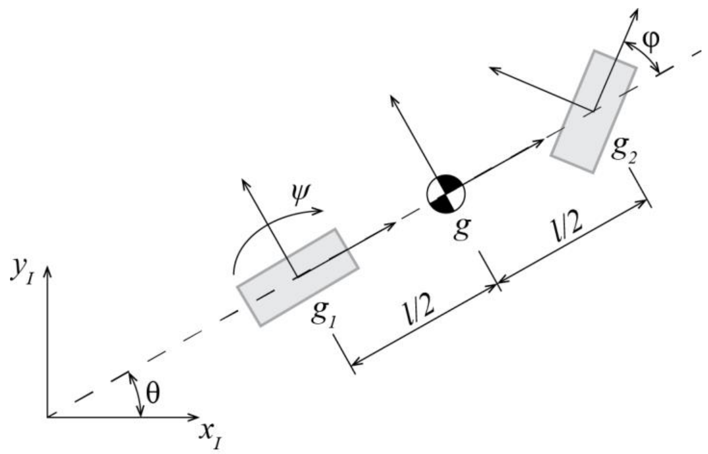

Fig. 2 Simplified model of Ackermann steered four wheeled vehicle

We can define nonholonomic constraints of the moving vehicle in inertial coordinate system or in vehicle's local coordinate systems which is easier. In local coordinate systems g_1 and g_2 these constraints are expressed as no-slip and no-slide conditions

$$\xi_{g_1}^{x} - r\dot{\psi} = 0, \tag{1}$$

$$\xi_{g_1}^{y} = 0, \tag{2}$$

$$\xi_{g_2}^{y} = 0, \tag{3}$$

where $\dot{\psi}$ is angular velocity of drive wheel, r is radius of wheel and ξ are velocities in local coordinate systems.

Position and orientation of local coordinate systems g_1 and g_2 relative to coordinate system g located at the vehicle's center of gravity is given by

$$\begin{aligned} g_{1,g} &= \left(-\tfrac{l}{2}, 0, 0\right), \\ g_{2,g} &= \left(\tfrac{l}{2}, 0, \varphi\right). \end{aligned} \tag{4}$$

Velocities expressed in local coordinate systems $\boldsymbol{\xi}_{g_1}$ and $\boldsymbol{\xi}_{g_2}$ can be mapped with velocities expressed in the vehicle's center of gravity $\boldsymbol{\xi}_g$. The map from vehicle's center of gravity to local velocities is the adjoint inverse action

$$\boldsymbol{\xi}_{g_1} = Ad^{-1}_{g_1,g}.\boldsymbol{\xi}_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{l}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \end{bmatrix} = \begin{bmatrix} \xi^x \\ \xi^y - \frac{l}{2}\xi^\theta \\ \xi^\theta \end{bmatrix},$$
$$\boldsymbol{\xi}_{g_2} = Ad^{-1}_{g_2,g}.\boldsymbol{\xi}_g = \begin{bmatrix} \cos\varphi & \sin\varphi & \frac{l}{2}\sin\varphi \\ -\sin\varphi & \cos\varphi & \frac{l}{2}\cos\varphi \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \end{bmatrix} = \begin{bmatrix} \xi^x\cos\varphi + \xi^y\sin\varphi + \xi^\theta\frac{l}{2}\sin\varphi \\ -\xi^x\sin\varphi + \xi^y\cos\varphi + \xi^\theta\frac{l}{2}\cos\varphi \\ \xi^\theta \end{bmatrix}. \tag{5}$$

Substituting velocity components $\xi^x_{g_1}$, $\xi^y_{g_1}$ and $\xi^y_{g_2}$ from equations (5) into nonholonomic constraints (1, 2, 3) we obtain differential equations in a Pfaffian form

$$\begin{aligned} &\xi^x - r\dot{\psi} = 0, \\ &\xi^y - \tfrac{l}{2}\xi^\theta = 0, \\ &-\xi^x\sin\varphi + \xi^y\cos\varphi + \xi^\theta\tfrac{l}{2}\cos\varphi = 0. \end{aligned} \tag{6}$$

The above equations we can rewrite into matrix form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{l}{2} \\ -\sin\varphi & \cos\varphi & \frac{l}{2}\cos\varphi \end{bmatrix} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \end{bmatrix} + \begin{bmatrix} -r & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{7}$$

By editing the above equation we obtain relation representing the influence of shape speeds acting on local velocity of the vehicle

$$\begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \end{bmatrix} = \begin{bmatrix} r & 0 \\ \frac{r}{2}\tan\varphi & 0 \\ \frac{r}{l}\tan\varphi & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\varphi} \end{bmatrix}. \tag{8}$$

In geometric mechanics, the equation of motion for principally kinematic system are generally expressed as kinematic reconstruction equation

$$\boldsymbol{\xi} = -\mathbf{A}(r)\dot{\mathbf{r}}, \tag{9}$$

where $\boldsymbol{\xi}$ is the velocity vector of the vehicle's center of gravity, $\mathbf{A}$ is the local connection associated with system constraints and $\dot{\mathbf{r}}$ is vector of the shape velocities.

Finally the mathematical model of the Ackermann steered vehicle expressed in inertial coordinate system is given by equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r & 0 \\ \frac{r}{2}\tan\varphi & 0 \\ \frac{r}{l}\tan\varphi & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} r\cos\theta - \frac{r}{2}\sin\theta\tan\varphi & 0 \\ r\sin\theta + \frac{r}{2}\cos\theta\tan\varphi & 0 \\ \frac{r}{l}\tan\varphi & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\varphi} \end{bmatrix}. \tag{10}$$

Conclusions

The aim of this paper was targeted to mathematical modelling of the four wheeled vehicle with nonholonomic constraints. Nonholonomic constraints were derived in coordinate system set in center of gravity of the vehicle by iterative Jacobian method. Based on these constraints was formed kinematic model of the vehicle.

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