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Analysis of Axially Compressed Cylindrical Shells Influenced by Geometrical Imperfections Using Finite Element Method

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Abstract. This article deals with the influence of imperfection on the resulting stiffness of the cylindrical shell. In the first step, it is pointed out that, the imperfection that is oriented towards middle of the shell has negative effect, because it reduces the magnitude of the critical force. The second part of the article deals with the influence of the length to imperfection and how this geometric dimension fundamentally affects the magnitude of critical force. The last part examines the imperfection in relation to the three defined force positions.

Introduction

Positive properties of shell elements are connected with their high strength with respect to their stiffness at minimum weight design. For this reasons they are used in designs, where steel structures have to have such basics properties, for example in planes, cranes, gas pipelines, etc. Design of thin-walled structures can be divided into two basic theories. Initial theory is considered to be linear theory, which considers the perfect design without imperfection. This approach can be used only for simple tasks. Nowadays, problem of nonlinear theory is developing for complicated technical tasks, where designers have to consider the imperfection.

However, this type of structure is considerably sensitive to different geometric variations, which in technical terms are called geometric imperfections. These imperfections have a major negative impact. Of their existence reduces the level of critical force that causes a loss of stability - buckling, as the authors indicated in [1-4]. The article deals with symmetrically cylindrical shell loaded by axial compressive load one end of which is fixed. The geometric imperfections are taken to be control parameters of our investigation with relation to the critical force magnitude. The computations are based on numerical calculations connected with linear buckling analysis.

Basic knowledge and problem presentation

This paper is focused to the modelling of geometrical imperfections of axial loaded cylindrical shells. According to publication [5] there are five different methods that can be used for establishing relations between geometrical imperfections and knock down factors. The methods are:

- Linear buckling mode-shaped imperfection (LBMI),
- Single perturbation load imperfection (SPLI),
- Geometrical dimple imperfection (GDI),
- Axisymmetric imperfections (ASI),
- Mid-surface imperfection (MSI).

For our case, we will consider GDI method. The displacement field of the GDI is given as radial displacement and it is defined as a dimple cosine with wavelengths along the circumference (a) and the meridian (b):

$$\Delta r(\varphi, \zeta) = \frac{w_0}{4}\left[1 - \cos\left(\frac{2\pi r}{a}\varphi\right)\right]\left[1 - \cos\left(\frac{2\pi}{b}\zeta\right)\right], \tag{1}$$

where:
- $w_0 \rightarrow$ imperfection amplitude [m],
- $a \rightarrow$ wavelength along the circumference [m],
- $b \rightarrow$ wavelength along the meridian [m],
- $r \rightarrow$ radius [m].

For numerical approach we can consider linear bifurcation analysis. Linear bifurcation analysis is carried out to obtain the elastic critical buckling resistance of the perfect structure. In numerical software, the elastic critical buckling resistance is obtained through the eigenvalue problem given in (2), where the load for which the stiffness matrix becomes singular is sought:

$$(\lambda_i \mathbf{K})\bar{v}_i = 0, \tag{2}$$

where:
- $\lambda_i \rightarrow$ eigenvalue (elastic critical buckling resistance),
- $\mathbf{K} \rightarrow$ tangent stiffness matrix,
- $\bar{v}_i \rightarrow$ buckling mode shapes (eigenvectors) [6].

Let's introduce a structural element for which the analysis was accomplished. The constant dimensions are:
- L (shell length) $\rightarrow$ L = 1 m,
- D (shell diameter) $\rightarrow$ D = 0.2 m.

For a better understanding of behaviour of the shell imperfections, the following variable parameters are chosen:
- $t_{\min}$ (minimal wall thickness) $\rightarrow t_{\min}$ = 0.5 mm,
- $t_{\max}$ (maximal wall thickness) $\rightarrow t_{\max}$ = 10 mm,
- i_t (increase increment for wall thickness) $\rightarrow i_t$ = 0.5 mm,
- $e_{\min}$ (minimal imperfection diameter) $\rightarrow e_{\min}$ = 0 mm,
- $e_{\max}$ (maximal imperfection diameter) $\rightarrow e_{\max}$ = 10 mm,
- i_e (increase increment for imperfection diameter) $\rightarrow i_e$ = 0.5 mm,

It is supposed that the structure is made of material with Young modulus E = 2.1e5 MPa and Poisson ratio μ = 0.3.

In order to have the most accurate data, the parametric model was created in MATLAB. To simplify work, the MATLAB script was created such a way that it directly generates data files for ABAQUS software which subsequently carried out numerical simulations. This resulted to more than 700 values that were evaluated.

Simulation of geometric imperfection for axisymmetric cylindrical shell

In this section the shell behaviour with relation to different magnitudes of geometric imperfections is described. The boundary conditions for FEM analyses are as follows. The shell is considered as a cantilever beam with one end fixed (removed all displacements and rotations) and the second one loaded by compression force of magnitude 1 N, fig. 1.

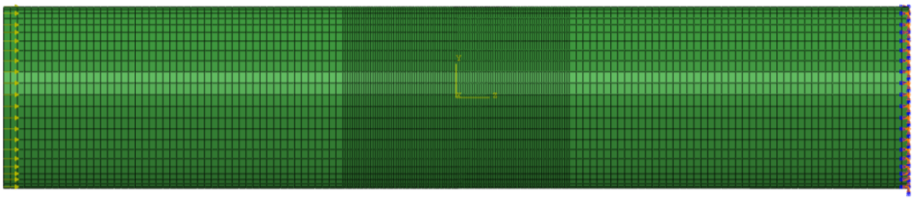

Fig. 1 Defined boundary condition for FEM analyses

At the beginning, we will change the shell diameter, so that it will rise to positive values and we will compare results with the state, when the shell diameter in location of imperfection decreases fig. 2. If the diameter of the shell at the location of imperfection decreases, decreases also critical force. From the numerical computations we see that the diameter decreasing leads to significant decreasing of the critical force F_{cr}, as shown in fig. 3a and fig 3b. For the case, when the diameter of shell at the location of imperfection increases, we can meet with obvious phenomenon that the magnitudes of critical force are growing

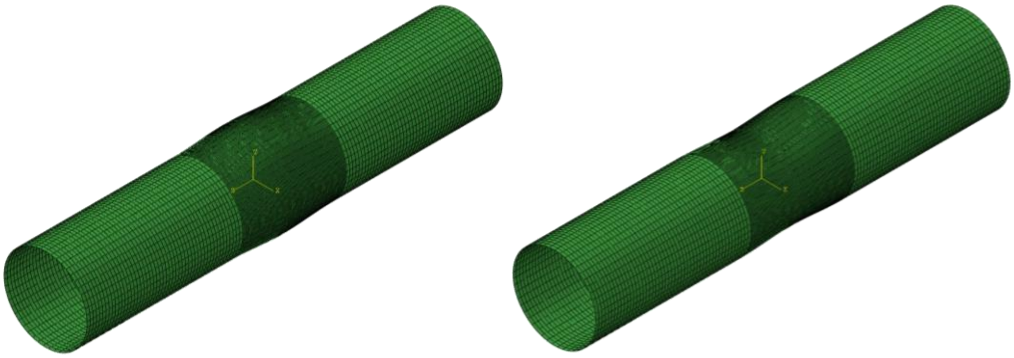

Fig. 2 Created imperfection

For further studies, we will only deal with cases, where the diameter at the location of imperfection decreases, because this leads to smaller critical forces.

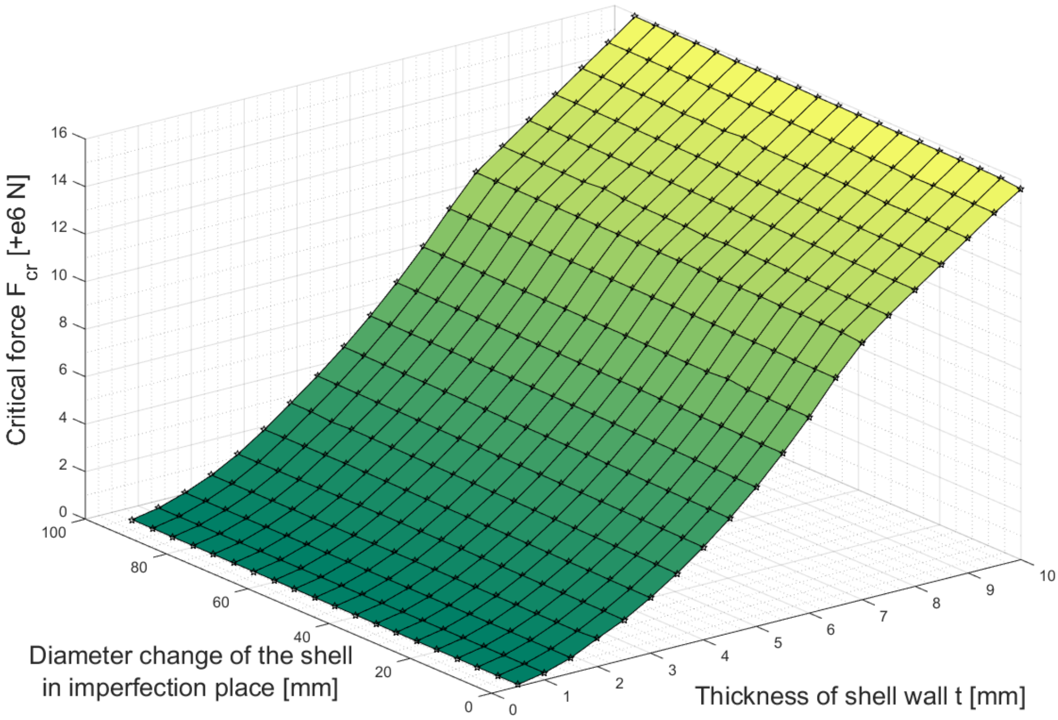

Fig. 3a Field values depending on the size of shell diameter and critical force

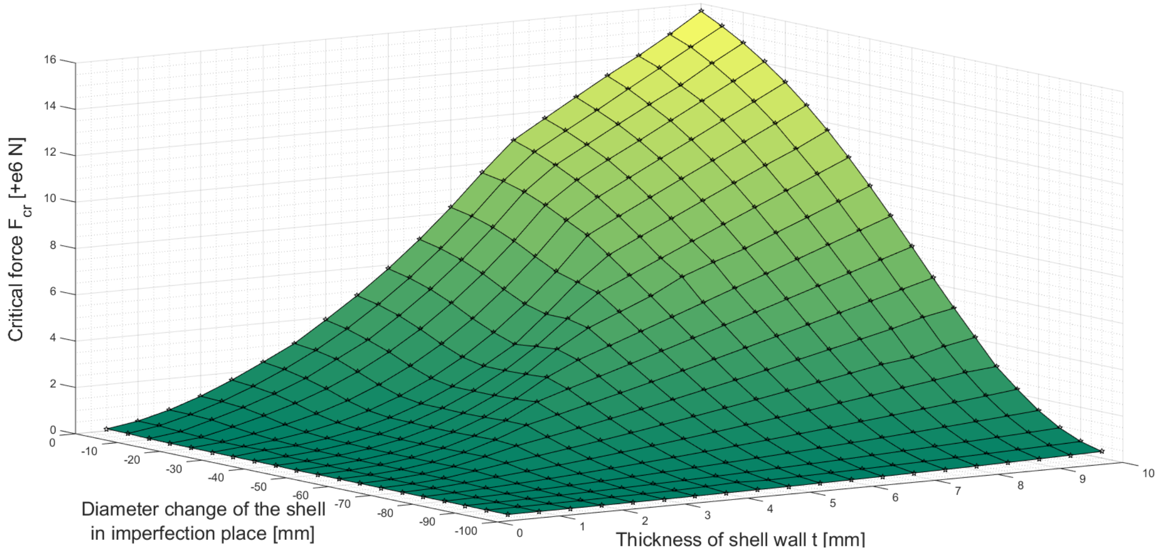

Fig. 3b Field values depending on the size of shell diameter and critical force

In the next step, we describe, how to length of imperfection affects the critical force F_{cr}. There were three models prepared with defined imperfection length:

- e_{l1} = 500 cm,
- e_{l2} = 400 cm,
- e_{l3} = 300 cm.

These imperfections were placement in geometrical centre of shell length. Since we can assume shell behaviour, therefore, it is not necessary to count with 400 values as previous calculations. The changes are relates to increase increment for wall thickness. The previous value is i_t = 0.5 mm and current is i_t = 1 mm. And last change in this case, is reduce the diameter range of imperfection e_d = 0.25; 0.5; 0.75; 1; 1.25; 1.5 mm. These dimensions were chosen because, if they were produced during technological manufacturing of the structural element and if in manufacturing process is absence of quality checking, such these damaged elements could be shipped to the seller. In the fig. 4a, 4b and 4c, shows the individual fields of dependence on the length of the imperfection e_l.

In the last step, we will examine shell behaviour, if we change the position of imperfection. Firstly was again changed variable parameter, due to faster numerical calculation. The change occurs diameter range imperfection actual values are e_d = 0.25; 0.5; 0.75; 1 mm and imperfection length is constant e_l = 250 cm.

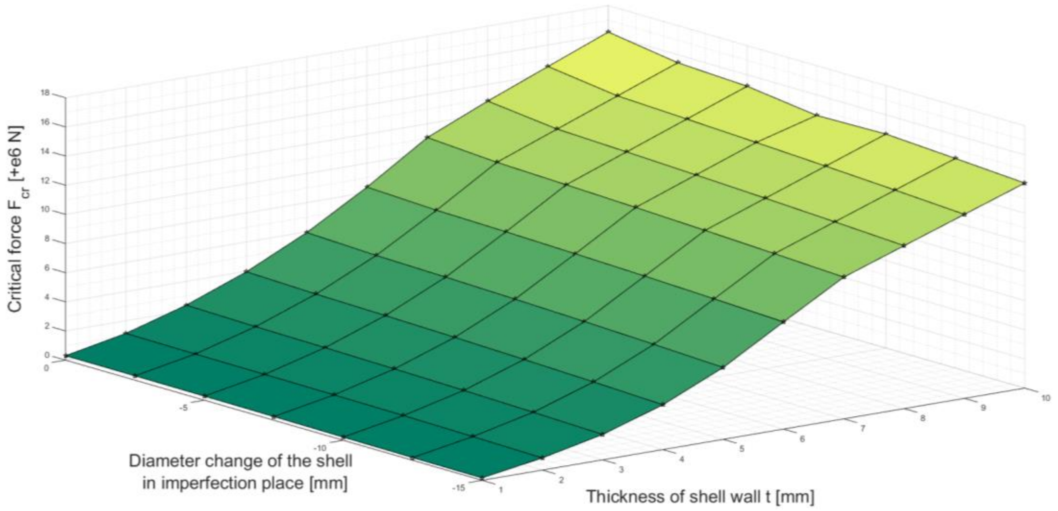

Fig. 4a Field of critical force in dependence on the length of imperfection e_{l1} = 500 cm

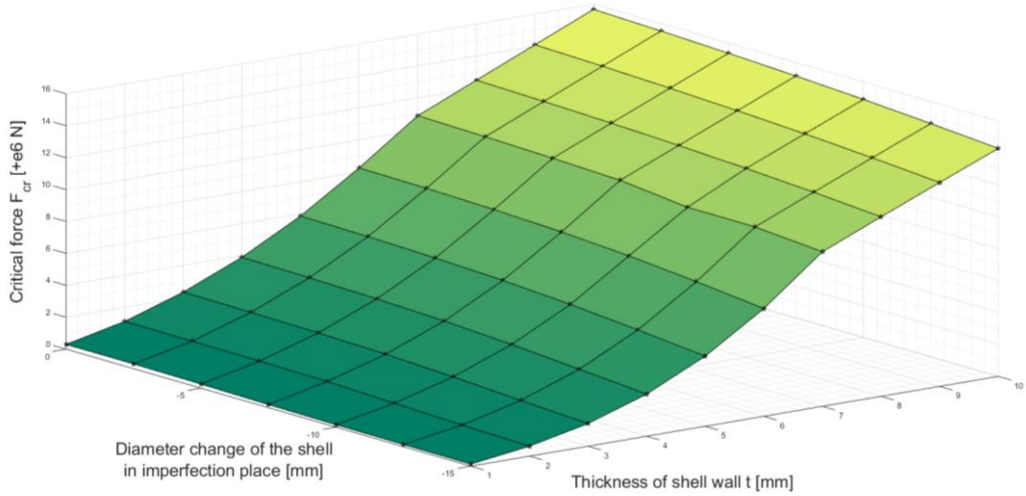

Fig. 4b Field of critical force in dependence on the length of imperfection e_{l1} = 400 cm

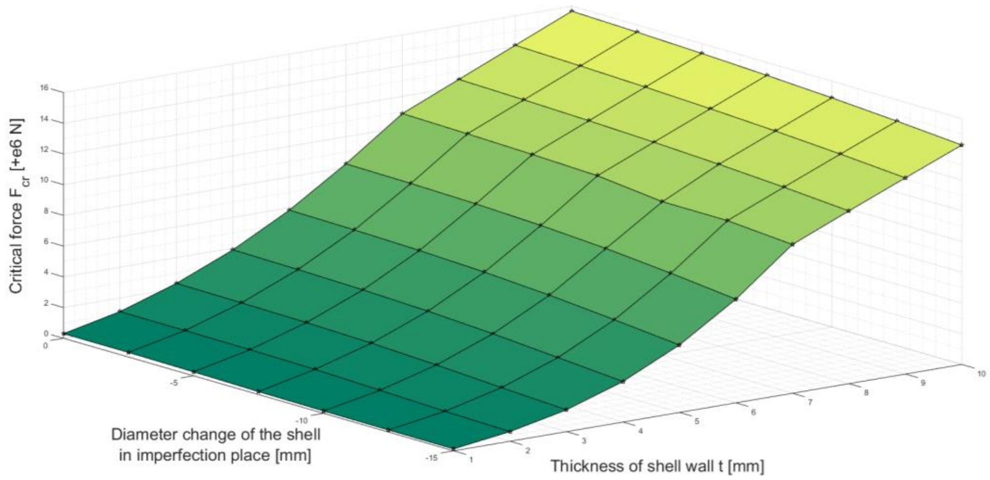

Fig. 4c Field of critical force in dependence on the length of imperfection e_{l1} = 300 cm

First position was placed on the symmetrical centre of the shell length. Second imperfection position is at a distance from the free end - 25 cm and the last position is at a distance from the fixed end - 25 cm. In the fig. 5a, 5b and 5c shows the individual fields of dependence on the positon of the imperfection.

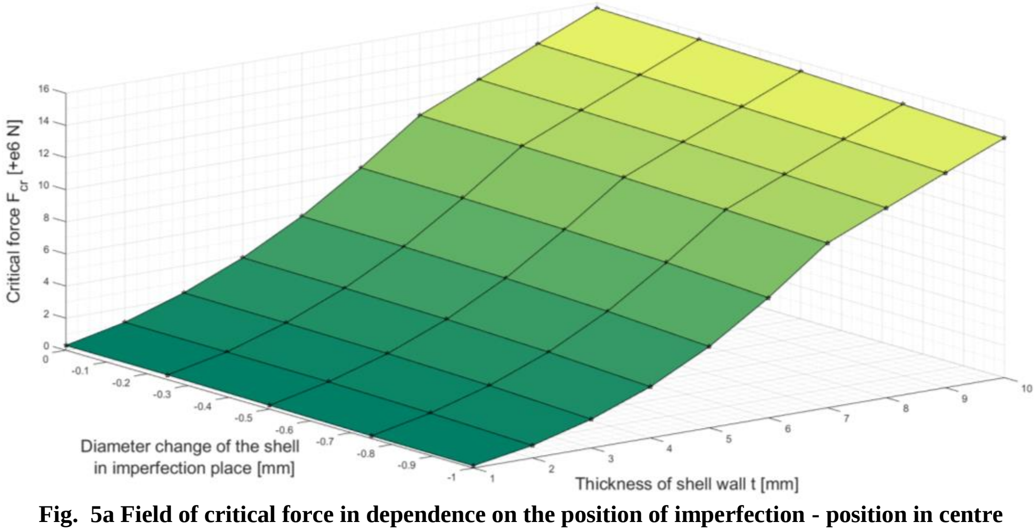

Fig. 5a Field of critical force in dependence on the position of imperfection - position in centre

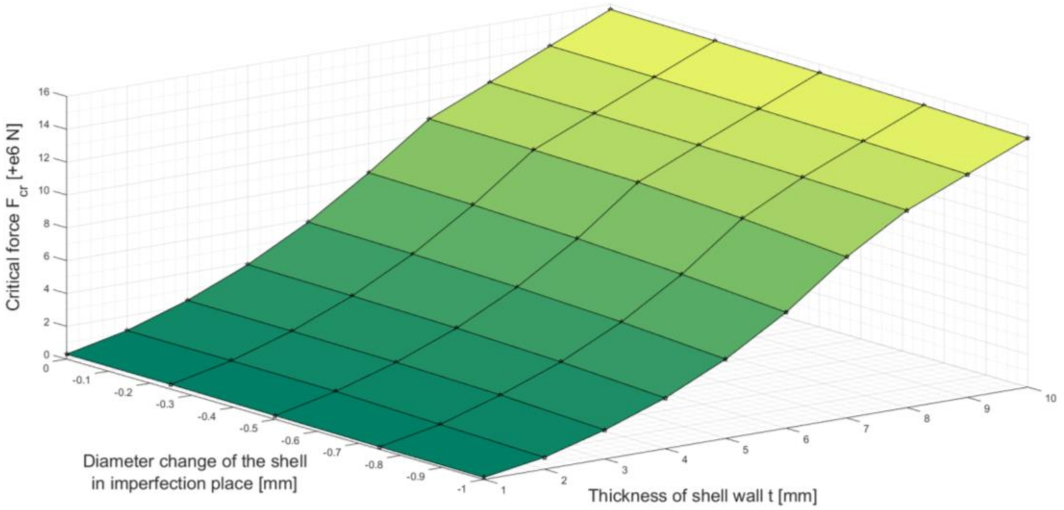

Fig. 5b Field of critical force in dependence on the position of imperfection - upper position

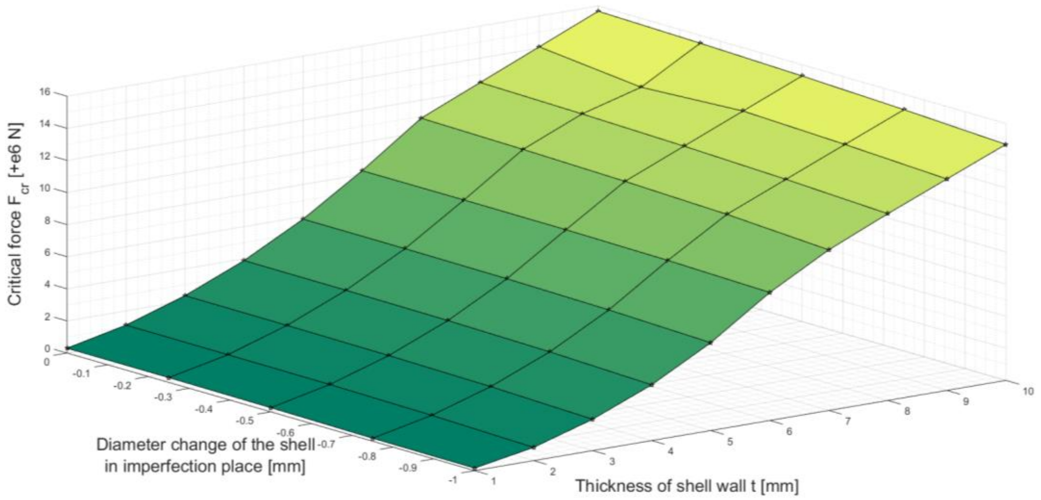

Fig. 5c Field of critical force in dependence on the position of imperfection - bottom position

Results

This chapter deals with evaluation of the results obtained from simulations, which were described in previous chapter. For first case, we can evaluate the imperfections that cause the tapering of the shell casing has a negative impact for critical force F_{cr}. But, an imperfection which causes the shell casing extension have positive effect on critical force $F_{cr,}$ for a better understanding see fig. 6.

We can conclude that the positive contribution of imperfection is manifested for wall thicknesses greater than 3 mm.

For second case fig. 7, we can observe, that the imperfection length significantly negative affects value of critical force for smaller wall thicknesses. For wall thicknesses greater than 7 mm, the imperfection length of imperfection does not have such a major impact for critical force.

For last case fig. 8, we can observe, when the imperfection is located in the part of shell, where is defined fixture, again for smaller wall thicknesses cause a decrease value of critical force.

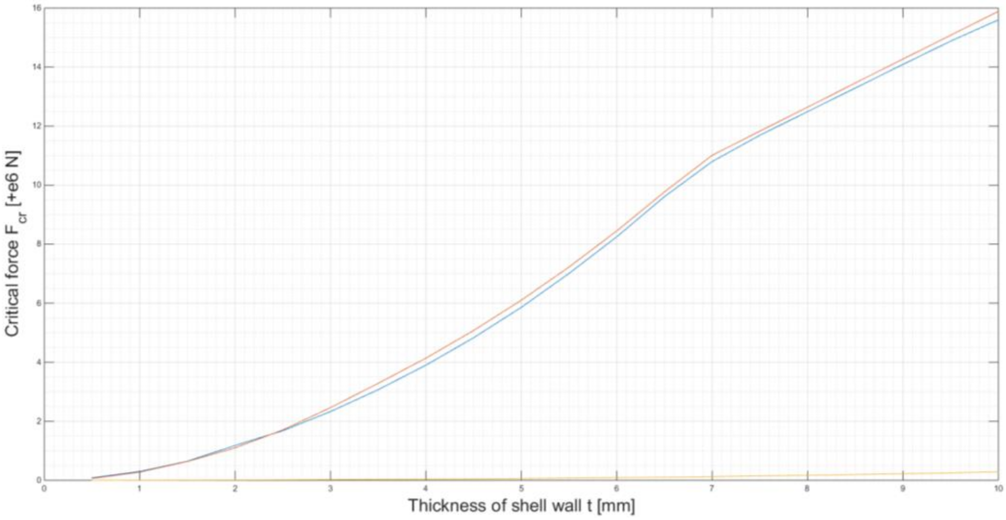

Fig. 6 The course of critical force at defined maximum eccentricity e_{max} = 95 mm, blue line – reference values without imperfection, red line – line with values + e_{max} , yellow line – line with values - e_{max}

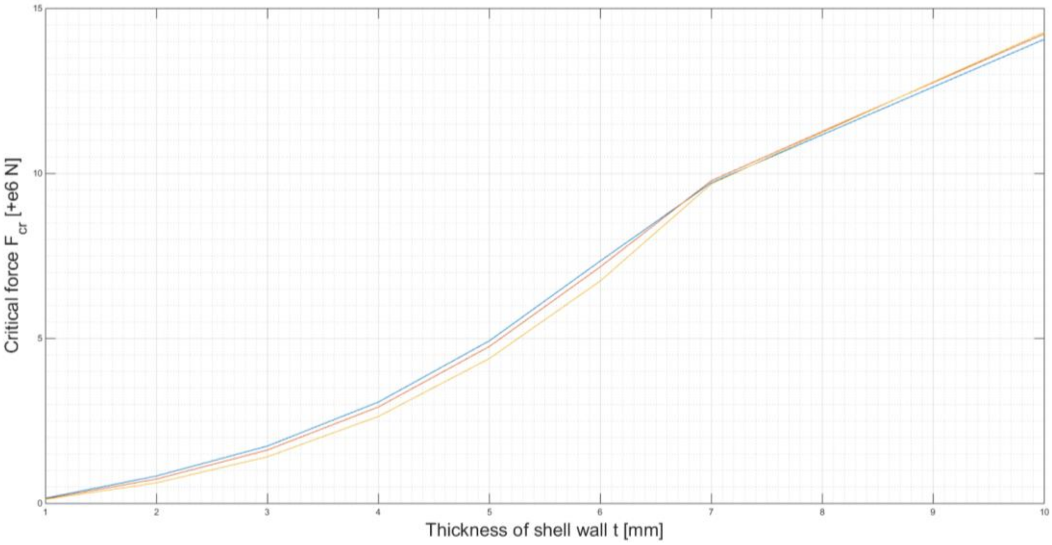

Fig. 7 The course of critical force influenced by the imperfection length, e_{max} = 1.5 mm, blue line → e_{l1} = 500 cm , red line → e_{l2} = 400 cm, yellow line → e_{l3} = 300 cm

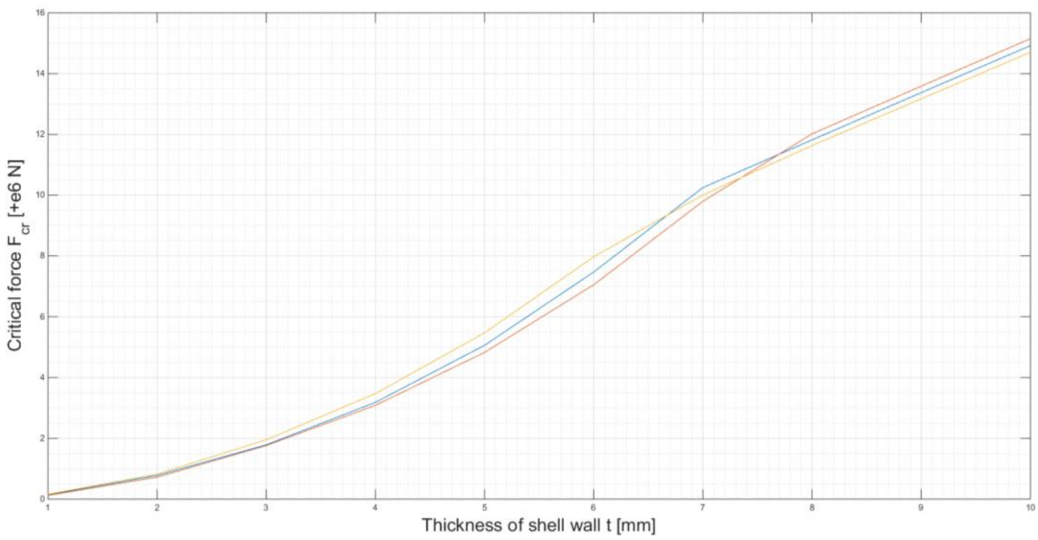

Fig. 8 The course of critical force influenced by the imperfection position, emax = 1.5 mm, blue line → e_{p1} = 0 cm , red line → e_{p2} = + 25 cm, yellow line → e_{p3} = - 25 cm

Conclusions

The article deals with the influence of geometrical imperfections for case of Axially Compressed Cylindrical Shells. In general, the issue of imperfection is considered to be an undesirable phenomenon, which was also confirmed by numerical calculations, when the value of critical force was reduces by the influence of geometric imperfections.

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