EAN 2017
55th conference on experimental stress analysis

An Alternative Material Model for Finite Strain J_2 Plasticity with Isotropic Hardening

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Keywords: finite-strain elastoplasticity, alternative J_2 material model, thermodynamically consistent formulation, finite element method

Abstract. In this paper an alternative J_2 material model with isotropic hardening for finite-strain elastoplasticity is presented. The model is based on a new non-linear continuum mechanical theory of finite deformations of elastoplastic media which allows us to describe the plastic flow in terms of various instances of the yield surface and corresponding stress measures in the body initial and current configurations. The approach also allows us to develop thermodynamically consistent material models in every respect. Consequently, the models not only do comply with the principles of material modelling, but also use constitutive equations, evolution equations and even 'normality rules' during return mapping which can be expressed in terms of power conjugate stress and strain measures or their objective rates. Therefore, such models and the results of the analyses employing them no longer depend on the description used in the model and the particularities of the material model formulation. Here we briefly present an improved version of our former material model capable of imitating ductile-to brittle failure mode transition at high strain rates in a ductile material and demonstrate the model in a numerical example.

Introduction

Modelling of materials within the framework of finite-strain elastoplasticity represents a challenging task in computational mechanics. While plastic behaviour of structural materials within the framework of small-strain elastoplasticity is now well understood, due to the fact, that small-strain flow plasticity theories work well and their results are in agreement with experiments, the same cannot be said about finite-strain flow plasticity theories [1]. Although innumerable material models for finite-strain elastoplasticity have by now been proposed [2-10], the models in general lack universality, as their analysis results depend on the description used in the model and the particularities of the model formulation. The modelling method might simply need some developments in the non-linear continuum theory of finite deformations of elastoplastic media in order that the corresponding material models could be considered to be complete and thermodynamically consistent.

There are two modelling methods or techniques in contemporary phenomenological plasticity to model irreversible finite deformations in the material of a deformable body. The first is an ad hoc extension of infinitesimal flow plasticity theories into the area of finite deformations of elastic media to cover large displacements, but small strains of the deforming body. The related material models use an additive decomposition of a strain rate tensor into an elastic part and a plastic part and are based on a hypoelastic stress-strain relationship while utilizing the theory of nonlinear continuum mechanics of elastic media [2, 11-16].

The second method is now generally accepted as a modelling method of finite irreversible deformations in the material. It is based on the theory of single-crystal plasticity which utilizes the micromechanics of irreversible deformations of a single-crystal to describe large plastic deformations in the material. The corresponding formulations introduce the notion of an intermediate stress-free configuration and use a multiplicative split of a deformation gradient into an elastic part and a plastic part, the theory of nonlinear continuum mechanics of elastic media and classical flow plasticity theories [2, 3, 17-23].

Though it may sound surprising, our ongoing research has shown, that both of the aforementioned methods are just variants of our nonlinear continuum theory of elastoplastic media using an additive decomposition of the displacement field into an elastic part and a plastic part which describes the plastic flow in terms of various instances of a yield surface and stress measures either in the body initial or current configuration. The theory also enables to develop thermodynamically consistent material models in every respect.

The first method using the additive split of the strain rate tensor is moreover constrained when the plastic flow is defined in terms of a Cauchy/Kirchhoff stress tensor based reference yield surface in the body current configuration. As a result, the corresponding plastic part of the spatial strain rate tensor does not have a finite-strain form. That is why contemporary material models based on the additive split of the strain rate tensor look, as if they had a mixed small-strain-finite-strain formulation.

The second method, employing the multiplicative split of the deformation gradient alone also opens a few questions when one considers the definition of the deformation gradient used with it. Since it neglects the elastic/plastic displacement field in the definition of the elastic/plastic part of the deformation gradient representing the factors of the multiplicative split, the formulation of the deformation gradient in this way seems to be rather incomplete and not quite consistent with the theory of nonlinear continuum mechanics. Moreover, it can be shown, that the theory using the multiplicative split would result in the same final formula for the deformation gradient as our theory if the displacement fields were properly considered in the multiplicative split.

We will not present the non-linear continuum theory of elastoplastic media herein, nor will we explore any of its part. Our aim in this paper is to present an alternative J_2 material model with isotropic hardening, which as a result of the aforementioned nonlinear continuum theory, allows us to develop thermodynamically consistent alternative material models.

The Kinematics of Deformation

In order to describe the kinematics of deformation of an elastoplastic media, we assume that the material/Lagrangian displacement field can additively be decomposed into an elastic part and a plastic part $\mathbf{u} = \mathbf{u}^{el} + \mathbf{u}^{pl}$. In this case neither the Green strain tensor $\mathbf{E} = 1/2 \cdot \left(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I} \right)$ nor the Almansi strain tensor $\mathbf{e} = 1/2 \cdot \left(\mathbf{I} - \mathbf{F}^{-T} \cdot \mathbf{F}^{-1} \right)$ has a decomposition into an elastic part and a plastic part, but an additive decomposition exists when one evaluates the objective time derivatives of the tensors. The material $\dot{\mathbf{E}}$ and the spatial $\mathbf{d} = \mathcal{L}_e \left(\mathbf{e} \right)$ strain-rate tensors then take the following forms

$$\dot{\mathbf{E}} = \frac{1}{2} \cdot \left(\dot{\mathbf{F}}^T \cdot \mathbf{F} + \mathbf{F}^T \cdot \dot{\mathbf{F}} \right) = \dot{\mathbf{E}}^{el} + \dot{\mathbf{E}}^{pl}, \tag{1}$$

where

$$\dot{\mathbf{E}}^{el} = \frac{1}{2} \cdot \left[\left(\frac{\partial \dot{\mathbf{u}}^{el}}{\partial \mathbf{X}} \right)^T \cdot \mathbf{F} + \mathbf{F}^T \cdot \frac{\partial \dot{\mathbf{u}}^{el}}{\partial \mathbf{X}} \right], \tag{2}$$

$$\dot{\mathbf{E}}^{pl} = \frac{\dot{\lambda}}{2} \cdot \left[\left(\frac{\partial^P \Psi}{\partial \mathbf{P}} \right)^T \cdot \mathbf{F} + \mathbf{F}^T \cdot \frac{\partial^P \Psi}{\partial \mathbf{P}} \right], \tag{3}$$

$$\frac{\partial \dot{\mathbf{u}}^{pl}}{\partial \mathbf{X}} = \dot{\lambda} \cdot \frac{\partial^P \Psi}{\partial \mathbf{P}}, \quad \text{and} \quad \frac{\partial^P \Psi}{\partial \mathbf{P}} \neq \left(\frac{\partial^P \Psi}{\partial \mathbf{P}} \right)^T, \tag{4}$$

$$\mathbf{d} = \mathbf{F}^{-T} \cdot \dot{\mathbf{E}} \cdot \mathbf{F}^{-1} = \mathbf{d}^{el} + \mathbf{d}^{pl}, \quad \mathbf{d}^{el} = \mathbf{F}^{-T} \cdot \dot{\mathbf{E}}^{el} \cdot \mathbf{F}^{-1}, \quad \mathbf{d}^{pl} = \mathbf{F}^{-T} \cdot \dot{\mathbf{E}}^{pl} \cdot \mathbf{F}^{-1}. \tag{5}$$

Here $\mathbf{X}$ denotes the position vector of a material point and $\mathbf{x} = \mathbf{X} + \mathbf{u}$ is the position vector of the corresponding spatial point after deformation. The deformation gradient $\mathbf{F} = \mathbf{I} + \partial \mathbf{u} / \partial \mathbf{X} = \mathbf{I} + \partial \mathbf{u}^{el} / \partial \mathbf{X} + \partial \mathbf{u}^{pl} / \partial \mathbf{X}$ then can either be expressed as a function of the material displacement field $\mathbf{u}$ alone or as a function of its elastic $\mathbf{u}^{el}$ and plastic $\mathbf{u}^{pl}$ parts. The symbols $\dot{\mathbf{E}}^{el}, \dot{\mathbf{E}}^{pl} / \mathbf{d}^{el}, \mathbf{d}^{pl}$ denote the elastic and plastic material/spatial strain rate tensors, in which the plastic flow is defined by Eqn. $(4)_1$ as a product of a plastic multiplier $\dot{\lambda}$ and an appropriate yield surface normal $\partial^P \Psi / \partial \mathbf{P}$ defined in terms of the 1st Piola-Kirchhoff stress tensor $\mathbf{P}$. Here the operator $\mathcal{L}_e(\bullet) = \mathbf{F}^{-T} \cdot \left[\partial \left(\mathbf{F}^T \cdot (\bullet) \cdot \mathbf{F} \right) / \partial t \right] \cdot \mathbf{F}^{-1}$ is the Lie derivative operator of a spatial strain tensor. Please note, that both the elastic and plastic parts of the strain rate tensor have similar forms as the strain rate tensor. Furthermore, it can be shown, that the plastic flow defined by Eqn. $(4)_1$ is not constrained, resulting in Eqns. (3) and $(5)_3$ respectively, as the only non-degenerated forms of the material and spatial plastic strain rate tensors.

The Constitutive Equation of the Material

Proper formulation of a material model for finite-strain elastoplasticity enables to define the constitutive equation of the material in terms of various stress and strain measures or their objective rates in both the body initial and current configurations. As a result, the constitutive equation of a material cannot be unique, but it must have various forms. These forms however have to comply with the principles of material modelling, particularly meet the requirements of material objectivity and moreover be thermodynamically consistent in order that they would defined the same material. Furthermore, because the additive decomposition defined by Eqns. (1), $(5)_1$ exists in rate forms only, the constitutive equation of the material too has to have a rate form. In fact, Eqns. (6)-(9) define a true hypoelastic based elastoplastic material model, which does not have a form in terms of a finite strain measure.

In this research we have modified our former material model capable of imitating ductile-to-brittle failure mode transition of a ductile material at high strain rates [24]. In agreement with the above, the rate form of the constitutive equation of the material can take any of the following forms:

$$\dot{\mathbf{S}} = {}^{mat}\mathrm{C}^{el} : \left(\dot{\mathbf{E}} - x \cdot \dot{\mathbf{E}}^{pl} \right) + {}^{mat}\mathrm{C}^{vis} : \left[\ddot{\mathbf{E}} - (1-x) \cdot \ddot{\mathbf{E}}^{pl} \right], \tag{6}$$

$$\mathcal{L}_P(\mathbf{P}) = \mathbf{F} \cdot \dot{\mathbf{S}} = \mathbf{F} \cdot \left\{ {}^{mat}\mathrm{C}^{el} : \left(\dot{\mathbf{E}} - x \cdot \dot{\mathbf{E}}^{pl} \right) + {}^{mat}\mathrm{C}^{vis} : \left[\ddot{\mathbf{E}} - (1-x) \cdot \ddot{\mathbf{E}}^{pl} \right] \right\}, \tag{7}$$

$$\mathcal{L}_O(\boldsymbol{\tau}) = \mathbf{F} \cdot \dot{\mathbf{S}} \cdot \mathbf{F}^T = J \cdot {}^{spat}\mathrm{C}^{el} : \left(\mathbf{d} - x \cdot \mathbf{d}^{pl} \right) + J \cdot {}^{spat}\mathrm{C}^{vis} : \left[\mathcal{L}_e(\mathbf{d}) - (1-x) \cdot \mathcal{L}_e\left(\mathbf{d}^{pl} \right) \right], \tag{8}$$

$$\mathcal{L}_T(\boldsymbol{\sigma}) = J^{-1} \cdot \mathbf{F} \cdot \dot{\mathbf{S}} \cdot \mathbf{F}^T = {}^{spat}\mathrm{C}^{el} : \left(\mathbf{d} - x \cdot \mathbf{d}^{pl} \right) + {}^{spat}\mathrm{C}^{vis} : \left[\mathcal{L}_e(\mathbf{d}) - (1-x) \cdot \mathcal{L}_e\left(\mathbf{d}^{pl} \right) \right], \tag{9}$$

where $${}^{mat}\mathrm{C}^{el} = 2 \cdot G \cdot \mathrm{I} + \lambda^{el} \cdot \mathbf{1} \otimes \mathbf{1}, \quad {}^{mat}\mathrm{C}^{vis} = 2 \cdot G^{vis} \cdot \mathrm{I} + \lambda^{vis} \cdot \mathbf{1} \otimes \mathbf{1}, \tag{10}$$

$$G=\frac{E}{2\cdot(1+\nu)},\ \lambda^{el}=\frac{\nu\cdot E}{(1+\nu)\cdot(1-2\cdot\nu)},\ G^{vis}=\frac{E^{vis}}{2\cdot(1+\nu^{vis})},\ \lambda^{vis}=\frac{\nu^{vis}\cdot E^{vis}}{(1+\nu^{vis})\cdot(1-2\cdot\nu^{vis})},\tag{11}$$

$${}^{spat}\mathrm{C}^{el}_{ijkl}=J^{-1}\cdot F_{im}\cdot F_{jn}\cdot F_{ko}\cdot F_{lp}\cdot {}^{mat}\mathrm{C}^{el}_{mnop},\tag{12}$$

$${}^{spat}\mathrm{C}^{vis}_{ijkl}=J^{-1}\cdot F_{im}\cdot F_{jn}\cdot F_{ko}\cdot F_{lp}\cdot {}^{mat}\mathrm{C}^{vis}_{mnop}.\tag{13}$$

In Eqns. (6)-(13) the symbols $\mathbf{S},\mathbf{P},\boldsymbol{\tau},\boldsymbol{\sigma},\dot{\mathbf{S}},\mathcal{L}_P(\mathbf{P}),\mathcal{L}_O(\boldsymbol{\tau}),\mathcal{L}_T(\boldsymbol{\sigma})$ denote the 2nd Piola-Kirchhoff stress tensor, the 1st Piola-Kirchhoff stress tensor, the Kirchhoff stress tensor, the Cauchy stress tensor and their objective rates respectively, namely the time derivative of the 2nd Piola-Kirchhoff stress tensor $\dot{\mathbf{S}}$, the Lie derivative of the 1st Piola-Kirchhoff stress tensor $\mathcal{L}_P(\mathbf{P})$, defined in terms of the Lie derivative operator of a mixed spatial-material stress tensor $\mathcal{L}_P(\bullet)=\mathbf{F}\cdot\left[\partial\left(\mathbf{F}^{-1}\cdot(\bullet)\right)/\partial t\right]$, the Oldroyd rate of the Kirchhoff stress $\mathcal{L}_O(\boldsymbol{\tau})$ defined in terms of the Lie derivative operator of a spatial stress tensor $\mathcal{L}_O(\bullet)=\mathbf{F}\cdot\left[\partial\left(\mathbf{F}^{-1}\cdot(\bullet)\cdot\mathbf{F}^{-T}\right)/\partial t\right]\cdot\mathbf{F}^{T}$ and the Truesdell rate of the Cauchy stress defined in terms of the Truesdell derivative operator of a spatial stress tensor $\mathcal{L}_T(\bullet)=J^{-1}\cdot\mathbf{F}\cdot\left[\partial\left(J\cdot\mathbf{F}^{-1}\cdot(\bullet)\cdot\mathbf{F}^{-T}\right)/\partial t\right]\cdot\mathbf{F}^{T}$, which actually carries out Lie differentiation, but with rearranged terms. Here the fourth order material elasticity tensor ${}^{mat}\mathrm{C}^{el}$ and the fourth order material viscosity tensor ${}^{mat}\mathrm{C}^{vis}$ have similar forms as the fourth order elasticity tensor of the St.-Venant Kirchoff material [25] using two independent material parameters E,ν and E^{vis},ν^{vis} respectively. The fourth order spatial elasticity and viscosity tensors ${}^{spat}\mathrm{C}^{el}$, ${}^{spat}\mathrm{C}^{vis}$ the can be determined in accordance with Eqns. (12) and (13), where $J=\det(\mathbf{F})$ is the Jacobian of the deformation. The variable x denotes the ratio of ductile and total damage increment [24]. Please also note, that the objective rates $\dot{\mathbf{S}},\mathcal{L}_P(\mathbf{P}),\mathcal{L}_O(\boldsymbol{\tau}),\mathcal{L}_T(\boldsymbol{\sigma})$ transform in the same way from one of their forms into another as do the stress tensors $\mathbf{S},\mathbf{P},\boldsymbol{\tau},\boldsymbol{\sigma}$, which ensure thermodynamic consistency of the formulation.

Modelling of the Plastic Flow

Similarly as in the case of the rate forms of the constitutive equation of the material, proper formulation of a finite-strain flow plasticity theory enables to describe the plastic flow in terms of various instances of a yield surface and corresponding stress measures in either the body initial or current configuration. Let the various instances of the yield surface ${}^{S}\Psi={}^{S}\Psi(\mathbf{S},\mathbf{q})$, ${}^{P}\Psi={}^{P}\Psi(\mathbf{P},\mathbf{q})$, ${}^{\tau}\Psi={}^{\tau}\Psi(\boldsymbol{\tau},\mathbf{q})$, ${}^{\sigma}\Psi={}^{\sigma}\Psi(\boldsymbol{\sigma},\mathbf{q})$ be defined in terms of the stress measures $\mathbf{S},\mathbf{P},\boldsymbol{\tau},\boldsymbol{\sigma}$ and a vector of internal variables $\mathbf{q}$. After introducing similar kinematic equations as the ones defined by Eqns. (1)-(5), they serve a basis for the first nonlinear continuum mechanical theory of finite deformations of elastoplastic media. Moreover, since they define the same admissible stress space and the same plastic flow respectively, the instances of the yield surface cannot be independent of each other. In fact ${}^{S}\Psi,{}^{P}\Psi,{}^{\tau}\Psi,{}^{\sigma}\Psi$ are related by the following formulas

$$\frac{\partial{}^{P}\Psi}{\partial\mathbf{P}}\cdot\mathbf{F}^{-1}=\frac{\partial{}^{\sigma}\Psi}{\partial\boldsymbol{\sigma}},\quad \frac{\partial{}^{P}\Psi}{\partial\mathbf{P}}\cdot\mathbf{F}^{-1}=\frac{\partial{}^{\tau}\Psi}{\partial\boldsymbol{\tau}},\quad \mathbf{F}^{T}\cdot\frac{\partial{}^{P}\Psi}{\partial\mathbf{P}}=\frac{\partial{}^{S}\Psi}{\partial\mathbf{S}}.\tag{14}$$

Furthermore, one of the yield surfaces ${}^{S}\Psi, {}^{P}\Psi, {}^{\tau}\Psi, {}^{\sigma}\Psi$ has to be chosen as a reference yield surface to define the material model. It can be shown, that when ${}^{\sigma}\Psi$ or ${}^{\tau}\Psi$ is chosen as a reference yield surface in the current configuration of the body, we recover the contemporary flow plasticity theories.

Crucial part in finite-strain material modelling is thermodynamic consistency of the material model formulation. It ensures, that the material model is independent of the description and the particularities of the model formulation. The thermodynamic consistency of the plastic flow then can be expressed as follows

$$\dot{\mathbf{E}}^{pl} : \dot{\mathbf{S}} \cdot dV_0 = \frac{\partial \dot{\mathbf{u}}^{pl}}{\partial \mathbf{X}} : \mathcal{L}_P\left(\mathbf{P}\right) \cdot dV_0 = \mathbf{d}^{pl} : \mathcal{L}_O\left(\boldsymbol{\tau}\right) \cdot dV_0 = \mathbf{d}^{pl} : \mathcal{L}_T\left(\boldsymbol{\sigma}\right) \cdot dv, \tag{15}$$

where dV_0 is an infinitesimal volume element in the body initial configuration and $dv = J \cdot dV_0$ is its spatial counterpart. In terms of the above one can prove, that Eqn. (15) has an equivalent form defined as

$$\frac{\partial {}^{S}\Psi}{\partial \mathbf{S}} : \dot{\mathbf{S}} = \frac{\partial {}^{P}\Psi}{\partial \mathbf{P}} : \mathcal{L}_P\left(\mathbf{P}\right) = \frac{\partial {}^{\tau}\Psi}{\partial \boldsymbol{\tau}} : \mathcal{L}_O\left(\boldsymbol{\tau}\right) = J \cdot \frac{\partial {}^{\sigma}\Psi}{\partial \boldsymbol{\sigma}} : \mathcal{L}_T\left(\boldsymbol{\sigma}\right), \tag{16}$$

which is known as the 'normality rule' and which defines a rate form of a thermodynamically consistent return mapping procedure. The result is of fundamental importance in computational mechanics as it states, how the plastic multiplier ought to be calculated during return mapping when finite-strain elastoplastic analysis is carried out.

The Reference Yield Surface

As it has been mentioned in the above, the choice of the reference yield surface determines the material model. As a result, alternative material models can be developed. In our research we have generalized the J_2 flow plasticity theory with isotropic hardening, where we used the ${}^{P}\Psi = {}^{P}\Psi\left(\mathbf{P}, \mathbf{q}\right)$, Eqn. (17) yield surface as the reference yield surface to define our material model. Please also note, that we have changed the definition of the ${}^{P}J_2\left(\mathbf{P}\right) = \mathbf{P} : \mathbf{P}$ invariant, which no longer bases on the deviatoric part of the 1st Piola-Kirchhoff stress tensor $\mathbf{P}$. This is because of the fact, that the 1st Piola-Kirchhoff stress tensor transforms under the change of the observer as $\mathbf{P}^{+} = \mathbf{Q}_R \cdot \mathbf{P}$ and the ${}^{P}J_2\left(\mathbf{P}\right)$ is the only invariant, which is not affected by the change, i.e. ${}^{P}J_2\left(\mathbf{P}\right) = {}^{P}J_2\left(\mathbf{P}^{+}\right)$, where $\mathbf{Q}_R$ is an arbitrary rotating tensor expressing the relative rotation of the coordinate systems of the observer with respect to the reference coordinate system. As a result, the yield surface is no longer a cylinder, but a sphere.

$${}^{P}\Psi = {}^{P}\sigma_{eq} - {}^{P}\sigma_y \leq 0, \tag{17}$$

$${}^{P}\sigma_{eq} = {}^{P}\sigma_{eq}\left(\mathbf{P}\right) = \sqrt{{}^{P}J_2\left(\mathbf{P}\right)} = \sqrt{\mathbf{P} : \mathbf{P}}, \qquad {}^{P}\sigma_y = F_{UT11} \cdot \left[\sigma_y + Q \cdot \left(1 - e^{-b \cdot e^{pl}}\right)\right], \tag{18}$$

$$\dot{e}^{pl} = \dot{e}^{pl}\left(\dot{\mathbf{F}}^{pl}\right) = \sqrt{\dot{\mathbf{F}}^{pl} : \dot{\mathbf{F}}^{pl}} = \dot{\lambda}, \;\; e^{pl} = \int_0^t \dot{e}^{pl} \cdot dt, \;\; \mathbf{F}^{pl} = \mathbf{I} + \frac{\partial \mathbf{u}^{pl}}{\partial \mathbf{X}}, \;\; \dot{\mathbf{F}}^{pl} = \frac{\partial \dot{\mathbf{u}}^{pl}}{\partial \mathbf{X}} = \dot{\lambda} \cdot \frac{\partial {}^{P}\Psi}{\partial \mathbf{P}}. \tag{19}$$

The actual yield stress ${}^{P}\sigma_y$, which is a 1st Piola-Kirchhoff stress measure, determines the radius of the yield surface and is defined by Eqn. $(18)_2$. It is the only nonzero component of a stress tensor $\mathbf{P}_{UT}$ (i.e. ${}^{P}\sigma_y = \left[\mathbf{P}_{UT}\right]_{11}$) coming from an uniaxial tensile test of the modelled

material, where the operator $\left[(\bullet)\right]_{11}$ extracts the element in the first row and the first column of a 2nd order tensor $(\bullet)$ written as a 3x3 square matrix. The corresponding deformation gradient and the Jacobian of deformation are denoted as $\mathbf{F}_{UT}, J_{UT}$, where $F_{UT11} = \left[\mathbf{F}_{UT}\right]_{11}$ and $J_{UT} = \det\left(\mathbf{F}_{UT}\right) = F_{UT11}$. Please also note, that the only nonzero element of the corresponding 2nd Piola-Kirchhoff stress tensor $\mathbf{S}_{UT}$ determined from the tensile test of the material is $\sigma_y + Q\cdot\left(1-e^{-b\cdot e^{pl}}\right) = \left[\mathbf{S}_{UT}\right]_{11}$. The equation is known as the NOIH rule of isotropic hardening [26], where $\mathbf{P}_{UT} = \mathbf{F}_{UT}\cdot\mathbf{S}_{UT}$. The NOIH rule uses three material parameters, the constant yield limit of the material σ_y and b, Q, which control the isotropic hardening. The parameters σ_y, Q are also 2nd Piola-Kirchhoff stress measures, while e^{pl} is an accumulated plastic strain Eqn. $(19)_2$. One may note here, that we have changed the definition of the accumulated plastic strain rate $\dot{e}^{pl}$ Eqn. $(19)_1$, where $\mathbf{F}^{pl}$ is the deformation gradient of pure plastic deformations in the material whose time derivative is assumed to be in the form of Eqn. $(4)_1$. Other changes in the definitions of the accumulated plastic strain rate $\dot{e}^{pl}$ and the equivalent stress ${}^{P}\sigma_{eq}$ resulted from the need of meeting the requirements of thermodynamic consistency in booth a one-dimensional (1D) stress state and a three-dimensional (3D) stress state in the material.

Plastic Multiplier Calculation

The calculation of the plastic multiplier is a crucial step in finite-strain elastoplastic stress analysis as it determines the value of the stress rate tensor Eqn. (6)-(9), and the plastic part of the strain rate tensors $\dot{\mathbf{E}}^{pl}, \mathbf{d}^{pl}$ during return mapping. The return mapping procedure moreover has to be thermodynamically consistent, i.e. it has to comply with Eqn. (16). The condition has not yet been met in any formulation so far. The thermodynamically consistent return mapping procedure for plastic multiplier calculation then utilizes the objective differentiation of the yield surface ${}^{P}\Psi$ and it can be expressed as follows

$$\frac{\partial {}^{P}\Psi}{\partial \mathbf{P}} : \mathcal{L}_{\mathrm{P}}\left(\mathbf{P}\right) - \left[\mathcal{L}_{\mathrm{P}}\left(\mathbf{P}_{UT}\right)\right]_{11} = 0, \tag{20}$$

where $\mathcal{L}_{\mathrm{P}}\left(\mathbf{P}\right)$ is then replaced by the rate form of the constitutive equation of the material Eqn. (7) and the other term of Eqn. (20) by the form $\left[\mathcal{L}_{\mathrm{P}}\left(\mathbf{P}_{UT}\right)\right]_{11} = F_{UT11}\cdot Q\cdot b\cdot e^{-b\cdot e^{pl}}\cdot\dot{e}^{pl}$. Please also note, that the first term of Eqn. (20) can be replaced with any other term of Eqn. (16) because of the thermodynamic consistency of the formulation.

The Ratio of Ductile and Total Damage Increment

The idea of the ratio of ductile and total damage increment x was first introduced by Écsi and Élesztős in order to take into account the internal damping of the material properly during plastic deformations, where x allowed us to distribute the plastic flow proportionally between the spring and the damper of a 1D frictional device representing the rheological model of the material [24]. The ratio is determined in an elastic corrector phase during return mapping and its value is then kept constant. Since the return mapping procedure in our material model is carried out in the 1st Piola-Kirchhoff stress space, we had to modify the definition of the ratio as follows

$$x = \frac{\left\langle \mathbf{N} : \mathbf{F} \cdot \left({}^{mat}\mathrm{C}^{el} : \dot{\mathbf{E}} \right) \right\rangle}{\left\langle \mathbf{N} : \mathbf{F} \cdot \left({}^{mat}\mathrm{C}^{el} : \dot{\mathbf{E}} \right) \right\rangle + \left\langle \mathbf{N} : \mathbf{F} \cdot \left({}^{mat}\mathrm{C}^{vis} : \ddot{\mathbf{E}} \right) \right\rangle}, \tag{21}$$

where

$$\frac{\partial^{P}\Psi}{\partial \mathbf{P}} = \mathbf{N}, \quad \mathbf{N} = \frac{\mathbf{P}}{\sqrt{\mathbf{P} : \mathbf{P}}} = \frac{\mathbf{P}}{\|\mathbf{P}\|}, \tag{22}$$

and

$$\langle y \rangle = \frac{y + |y|}{2} \geq 0 \tag{23}$$

denote the McCauly's brackets, which return zero if $y < 0$ and where we also used the property $\mathcal{L}_P(\mathbf{P}) = \mathbf{F} \cdot \dot{\mathbf{S}}$. Please also note that all terms of Eqn. (21) are objective stress rates, so that the value of x is not affected by the change of the observer.

Numerical Experiment

As a numerical experiment, plastic bending of a cantilever of dimensions 50mm x 50mm x 600mm has been studied. One third of the upper surface of the beam was loaded applying $p = 5.0$ MPa constant pressure on it as stepped load using a Heaviside step function using. The beam was initially at rest and the analysis was run as transient-dynamic using $0.5 \cdot 10^{-4}$ s time step size and 0.048 s calculation end time. Figure 3 depicts the spatially discretized beam, the boundary conditions and the applied pressure with arrows.

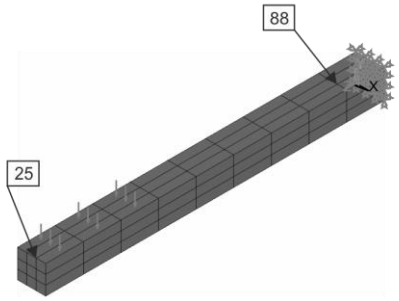

Fig. 1. Spatially discretized model of the beam

The cantilever material was a low carbon steel whose material properties are listed in Tab. 1.

Table 1. Material properties of the cantilever in the body initial configuration

E [Pa]	$2.1 \cdot 10^{11}$
E^{vis} [$Pa \cdot s$]	$2.1 \cdot 10^{7}$
$\nu = \nu^{vis}$[–]	0.3
σ_y [Pa]	$350.0 \cdot 10^{6}$
Q [Pa]	$150.0 \cdot 10^{6}$
b [-]	3.0
ρ_0 [kg/m^3]	7800.0

In order to simplify the analysis, we assumed, that $\mathbf{F}_{UT} = \mathbf{I}$ and $F_{UT11} = 1$, since we did not have information about the deformation gradient $\mathbf{F}_{UT}$ and its Jacobian J_{UT} coming from the tensile test of the material.

Numerical Results

Figures 2-5 show a few selected results of the finite element analysis, namely the Von-Mises stress distribution at the end of the analysis, the vertical displacement time history curve at nodes N25 and N88, the vertical velocity and the vertical acceleration time history curves at node N25 (see Fig. 1 for the location of the nodes). As can be seen in the figures, the loading of the body is highly dynamic with maximum deformations around 0.12 m, maximum velocity around 18.0 m.s^{-1} and maximum acceleration around 18500.0 m.s^{-2}.

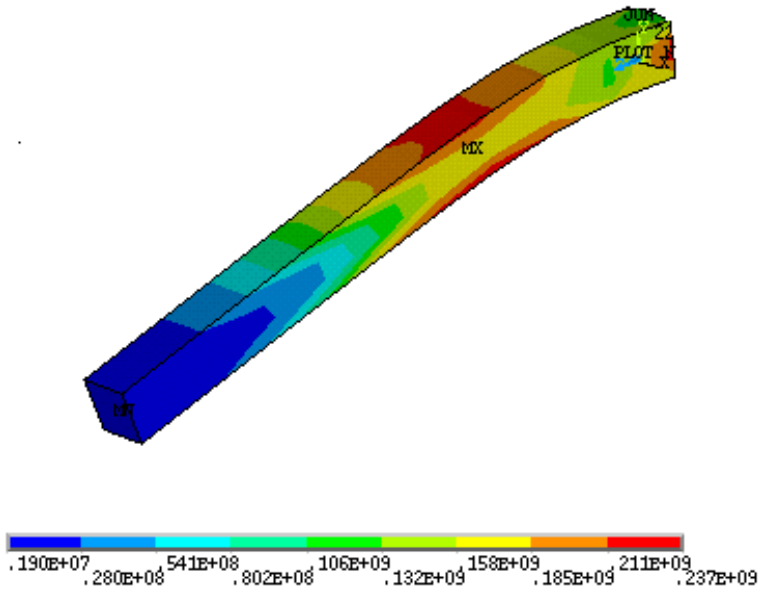

Fig. 2. Von Mises stress distribution at the end of the analysis

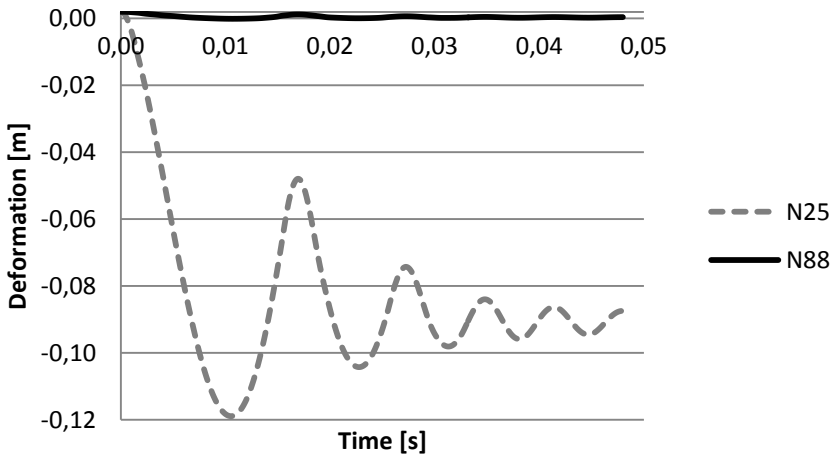

Fig. 3. Vertical deformation time history curves at nodes N25 and N88

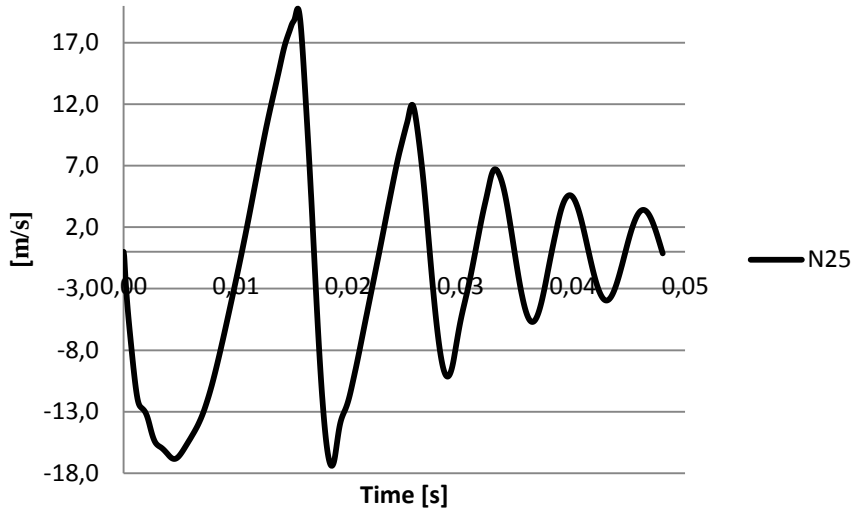

Fig. 4. Vertical velocity time history curve at node N25

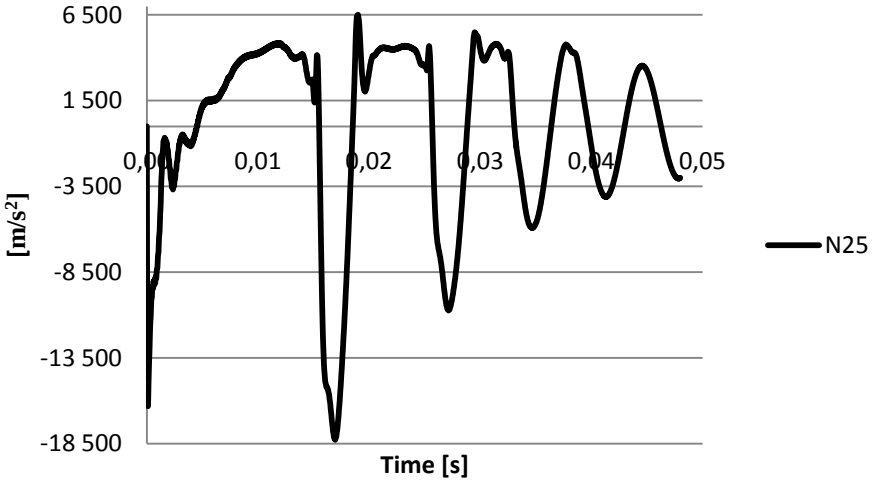

Fig. 5. Vertical acceleration time history curve at node N25

Conclusions

In this paper some recent developments in the solution of structural finite element problems within the framework of finite-strain elastoplasticity was presented. An alternative J_2 material model using an improved additive decomposition of the strain rate tensor into an elastic part and a plastic part, capable of modelling ductile-to-brittle failure mode transition of a ductile material at high strain rates, was developed. The model is a result of the first non-linear continuum theory of finite deformations of elastoplastic media which allows to describe the plastic flow in terms of various instances of the yield surface and corresponding stress measures in the body initial and current configurations. The model moreover is thermodynamically consistent. Therefore, it uses constitutive equations, evolution equations and even "normality rules" which can be expressed in terms of power conjugate stress and strain measures or their objective rates either in the body initial or current configuration. Consequently, the analysis results of such a model, are independent of the description used in the model and the particularities of the model formulation.

Acknowledgement

This publication is the result of the project implementation: Research of friction stir welding (FSW) application as an alternative to melting welding methods no. 26240220031 supported by the Research & Development Operational Programme funded by the ERDF. Funding from the VEGA grant 1/0740/16 resources is also greatly appreciated.

References

[1] G.A. Maugin, The Thermomechanics of Plasticity and Fracture, Cambridge University Press, Cambridge, 1992.

[2] J.C. Simo, T.J.R. Hughes, Computational Inelasticity, Springer, New York, 1998.

[3] E.A. de Souza Neto, D. Perić, D.R.J. Owen, Computational Methods for Plasticity, Theory and Applications, John Wiley & Sons Ltd., Singapore, 2008.

[4] S. Nemat-Nasser, Plasticity, A Treatise on Finite Deformation of Heterogenous Inelastic Materials, Cambridge University Press, Cambridge, 2004.

[5] A. Bertran, Elascity and Plasticity of Large Deformations, An Introduction, 3rd ed., Spinger, Berlin, Heidelberg, 2013.

[6] D.R.J. Owen, E. Hinton, Finite Elements in Plasticity, Theory and Practice, Pineridge Press, Swansea, 1980.

[7] T. Belytschko, W. K. Liu, B. Moran, Nonlinear Finite Elements for Continua and Structures, John Wiley & Sons LTD, Chichester, 2000.

[8] M.A. Crisfield, Non-linear Finite Element Analysis of Solids and Structures, Essentials, Vol. 1, John Wiley & Sons Ltd., Chichester, 2000.

[9] M.A. Crisfield, Non-linear Finite Element Analysis of Solids and Structures, Advanced Topics, Vol. 2, John Wiley & Sons LTD, Chichester, 2001.

[10] K.J. Bathe, Finite Element Procedures, Prentice-Hall INC., Englewood Cliffs, New Jersey, 1995.

[11] C. Truesdell, R.A. Toupin, The Classical Field Theories, Handbook der Physic, III/1, Springer-Verlag, Berlin, 1960.

[12] C. Truesdell, W. Noll, The Nonlinear Field Theories, Handbook der Physic, III/3, Springer-Verlag, Berlin, 1965.

[13] M.E. Gurtin, An Introduction to Continuum Mechanics, Academic Press, Orlando, 1981.

[14] J.E. Marsden, T.J.R. Hughes, Mathematical Foundations of Elasticity, Dover, New York, 1994.

[15] R.W. Ogden, Non-linear Elastic Deformations, Ellis Horwood Ltd., West Sussex, 1984.

[16] P. Cairlet, Mathematical Elasticity. Vol. 1, Three-dimenasional elasticity, North-Holland Publishing Company, Amsterdam, 1988.

[17] R.J. Asaro, Micromechanics of Crystals and Polycrystals, Advances in Applied Mechanics, Vol. 23., Academic Press, New York, 1983..

[18] G.I. Taylor, Analysis of Plastic Strain in a Cubic Crystal, in Stephen Timoshenko 60th aniversary volume, ed., J.M. Lessels, Macmillan, New York, 1938.

[19] R. Hill, Generalized constitutive relations for incremental deformation of metal crystals by multislip, Journal of Mechanics and Physics of solids, 14 (1966) 95-102.

[20] R. Hill, J.R. Rice, Constitutive analysis of elastic-plastic crystals at arbitrary strain, Journal of Mechanics and Physiscs of Solids, 20 (1972) 401-413.

[21] R.J. Asaro, J.R. Rice, Strain localization in ductile single crystals, Journal of Mechanics and Physiscs of Solids, 25 (1977) 309-338.

[22] R.J. Asaro, Geometrical effects in the inhomogeneous deformation of ductile single crystals, Acta Metallurgica, 27 (1979) 445-453.

[23] C. Sansour, J. Bocko, On the Numerical Implications of Multiplicative Inelasticity with an Anisotropic Elastic Constitutive Law, Int. J. Num. Meth. Engrg., 5, 14 (2003) 2131-2160.

[24] L. Écsi, P. Élesztős,: Moving Toward a More Realistic Material Model of a Ductile Material with Failure Mode Transition. "Mat.-wiss. u. Werkstofftech", 5, 43 (2012) 379-387.

[25] J. Bonet, R.D. Wood, Nonlinear Continuum Mechanics for Finite Element Analysis, 2nd ed., Cambridge University Press, Cambridge 2008.

[26] J. Lemaitre, Handbook of Material Behaviour Models, Vol. 1, Deformation of Materials, Academic press, London, 2001.