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Experimental Construction of Lamb Wave Dispersion Curves in Plates

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Abstract: The main aim of presented paper is experimental obtaining of Lamb wave dispersion curves in plates using broadband ultrasonic and acoustic emission transducers with additional converters facilitating the measurement of wavelength. The paper contains procedures of experimental investigation of phase as well as group velocity of the particular modes of Lamb wave and presents findings related to the effect of geometry on properties of particular propagation modes of Lamb wave.

Introduction

Lamb waves have a great deal of interest, especially for flaw detection, material characterization and, last but not least, for inspection of various types of layered structures [1, 2]. Lamb waves are one of the type of guided waves, which can exist only in geometries with finite thickness such as plates, rods or tubes [3]. The excitation and detection of Lamb waves can be realized by a variety of methods such as wedge method or comb-structure method [4] with incorporated piezoelectric transducers. Less frequent, in terms of Lamb wave excitation, is use of optical methods or incorporation of electromagnetic acoustic transducers (EMAT) [5].

Lamb waves exhibit dispersion during their propagation. Dispersion describes a relationship between the phase and group velocity with the frequency. The effect of dispersion is change of overall pulse shape in time domain with related changes in frequency domain. Dispersion can be caused by geometry and/or by viscoelasticity of the material [6]. The dispersion originating from geometry can be described by, so called, dispersion curves, which can be obtained by numerical solving of the Rayleigh-Lamb frequency equation [7].

The aim of this paper is to experimentally determine the phase and as well as group velocity of antisymmetric and symmetric mode of Lamb wave on sheet metals (steel grade 11) of various thicknesses and compare obtained results with numerically computed results. The discussion also contains findings related to propagation characteristics of antisymmetric and symmetric modes of Lamb wave excited in plates of various thicknesses.

Theory of Lamb waves – solution by method of potentials [7]

Consider plane strain problem of a free traction force surfaces as depicted on Fig. 1. The motion is defined as two-dimensional so that all quantities are independent on y coordinate. The unknown displacement vector $\bar{u}$ will be defined with use of Helmholtz decomposition theorem as follows:

$$\bar{\boldsymbol{u}} = \nabla\boldsymbol{\phi} + \nabla \times \bar{\boldsymbol{\psi}}, \tag{1}$$

where $\phi = \phi(x,z)$ is a potential function and $\bar{\psi} = (0, -\psi(x,z), 0)$ is a vector function, both of them are dependent on x and z coordinates.

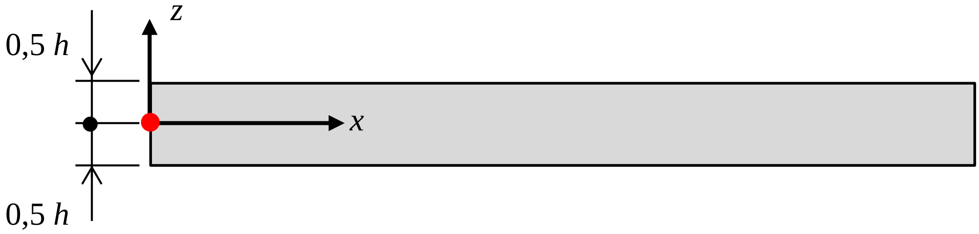

Fig. 1: Geometry of the plate; h is a thickness of the plate

The equation of motion expressed in terms of displacements can be with use of decomposed displacement vector separated into two wave equation, which represent governing equations for longitudinal (Eq. 2) and transversal waves (Eq. 3):

$$\nabla^2\phi - \frac{1}{c_{\mathrm{L}}^2}\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} = 0, \tag{2}$$

$$\nabla^2\psi - \frac{1}{c_{\mathrm{T}}^2}\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2} = 0, \tag{3}$$

where c_L is the speed of dilatational waves, c_T is the speed of shear waves. The assumed solutions of Eq. 2, resp. 3 for wave propagating from left to right are:

$$\phi = (K_1 \sin(pz) + K_2 \cos(pz))e^{\mathrm{i}(kx-\omega t)}, \tag{4}$$

$$\psi = (K_3 \sin(qz) + K_4 \cos(qz))e^{\mathrm{i}(kx-\omega t)}, \tag{5}$$

where $p^2 = \frac{\omega^2}{c_L^2} - k^2$, $q^2 = \frac{\omega^2}{c_T^2} - k^2$, K_1, K_2, K_3, K_4 are constants resulting from boundary conditions, ω is the angular frequency, k is wavenumber, t is a time variable and i is the imaginary unit. Expressing the displacements u (axis x) and w (axis z) as well as the stresses σ_{zz} and σ_{zx} in terms of potentials and applying a boundary condition $\sigma_{zx} = \sigma_{zz} = 0$ at $z = \pm\, 0{,}5\, h$ it is then possible to derive the Rayleigh-Lamb frequency equation for symmetric (Eq. 6) and antisymmetric (Eq. 7) modes:

$$\frac{\tan(\frac{qt}{2})}{\tan(\frac{pt}{2})} = -\frac{4k^2pq}{(q^2-k^2)^2}, \tag{6}$$

$$\frac{\tan(\frac{qt}{2})}{\tan(\frac{pt}{2})} = -\frac{(q^2-k^2)^2}{4k^2pq}. \tag{7}$$

Equations 6 and 7 are further used to obtain the dispersion curves for various modes of symmetric and antisymmetric Lamb wave, namely the dependence between phase velocity and the frequency-thickness product.

The dispersion curves for group velocity c_g as the function of frequency-thickness product can be derived from phase velocity dispersion curves using following definition [7]:

$$c_g = \frac{d(kc_p)}{dk} = c_p + k\frac{dc_p}{dk}. \tag{8}$$

Experimental measurement of dispersion curves

The experiments were carried out with the use of AMSY-6 acoustic emission system, manufactured by Vallen Systeme GmbH. The acoustic emission system in itself has been utilized as signal generator and power supply for Vallen AEP5 amplifiers with 34 dB gain (See Fig. 2).

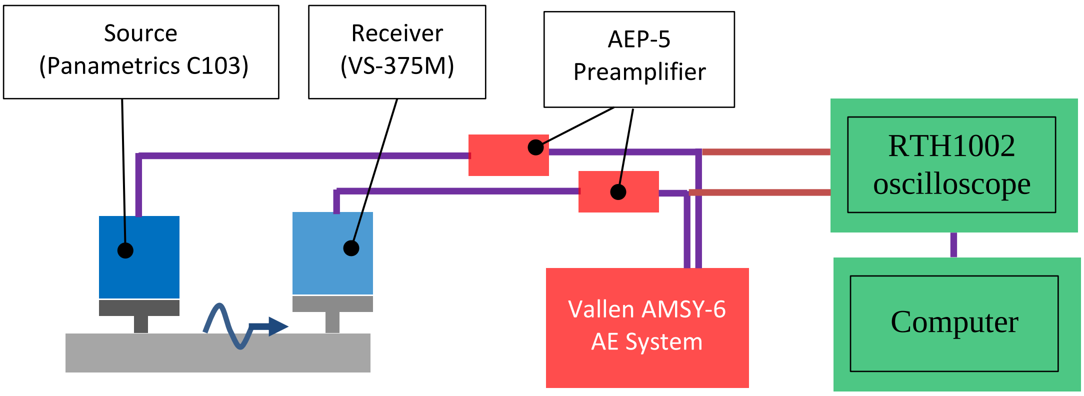

Fig. 2: Schematic representation of measurement system

A five-cycle sinusoidal tone burst of 250 $V_{peak\text{-}peak}$ and central frequency of 150 kHz (Fig. 3) has been used as driving signal for the broadband transducer Panametrics C103 1 MHz/0.5”, which figured as the source. The signal from propagating waves has been captured by broadband acoustic emission transducer Vallen VS375-M and amplified by Vallen AEP5 amplifier. The signal has been further processed with use of Rohde&Schwartz RTH1002 oscilloscope.

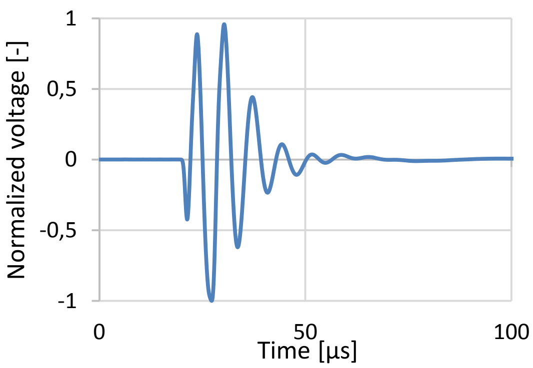

Fig. 3: Driving signal for wave excitation

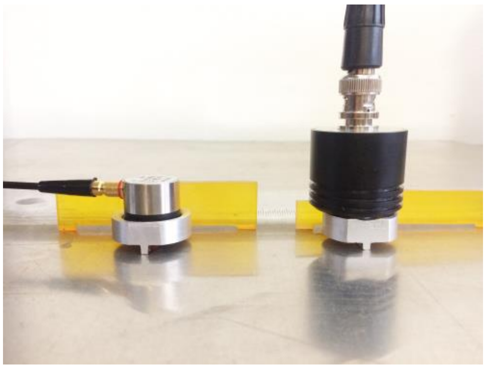

Fig. 4: Experimental setup for wavelength measurement

Both transducers were placed on self-manufactured converters (see Fig. 4), which have been mounted onto skids in order to provide sliding on metal sheet including precise measurement of mutual distance between transducer and receiver. The phase velocity has been measured according to principle, depicted in Fig. 5. Namely, the phase difference between points A and B corresponds to 2π, which implicitly results in the offset by the value of one wavelength [4, 7, 8]. The phase velocity can be therefore calculated from following expression:

$$c_p = \lambda f \tag{9}$$

where λ denotes wavelength and f denotes frequency of the particular part of the pulse, which wavelength is being measured.

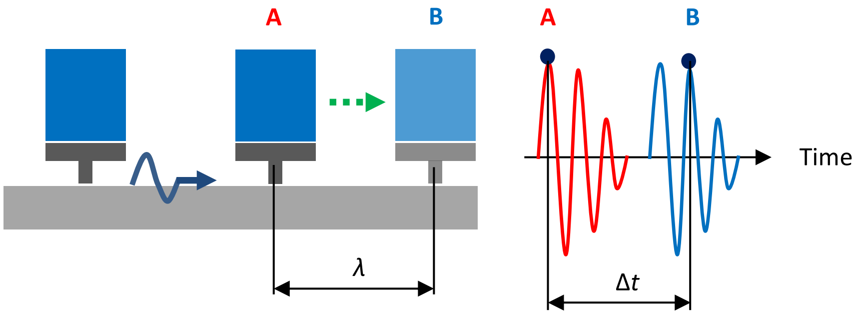

Fig. 5: Principle of the phase velocity measurement; Δt is the time delay

In case of the group velocity measurement, there has to be used somewhat different approach. The group velocity can be understood as the velocity of energy transportation [7]. In practice, we are measuring the time delay Δt corresponding to the distance Δd between positions A and B. It has to be noted, that the measurement of the time delay is related to the position of the maximum amplitude of the pulse (see Fig. 6) [7, 8].

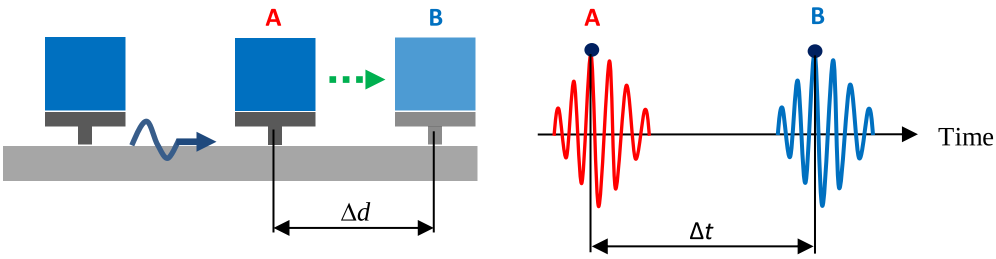

Fig. 6: Principle of the group velocity measurement

The group velocity can be therefore calculated as:

$$c_g = \frac{\Delta d}{\Delta t}. \tag{10}$$

Results and discussion

The phase as well as group velocities were measured on sheet metals (steel grade 11) of thicknesses 1, 2, 3, 4 and 6 millimetres with dimensions ranging from 40x30 cm to 100x70 cm. In case of phase velocity measurement, there has to be earmarked one period of the pulse, which would best possible characterize the entire symmetric (S0) and antisymmetric (A0) Lamb wave mode (see Fig. 7 – orange segment). From chosen period was computed the frequency and according to the procedure illustrated in Fig. 5 was measured the wavelength and finally computed (by Eq. 9) the phase velocity.

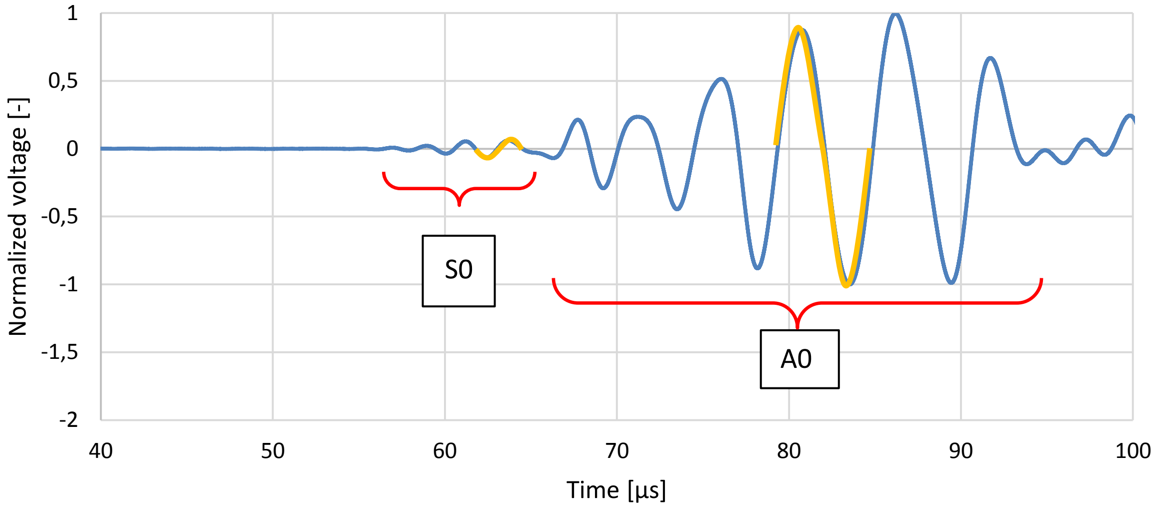

Fig. 7: A0/S0 mode separation during measurement of group and phase velocity on 1mm thick sheet metal

The group velocity measurement has been then realized according to the time delay versus propagation distance of the maximum pulse amplitude (see Fig. 6). Measurement of phase and group velocities of A0 flexural mode was easier to implement thanks to its much larger displacements in *z* axis compared to S0 mode within the measured frequency-thickness range. Figs. 8 and 9 display comparison between numerically and experimentally obtained dispersion curves. The numerically obtained dispersion curves were being calculated according to the Eqs. 6 and 7 with use of self-developed code in MATLAB software. The material properties were as follows: c_L = 5900 m/sec, c_T = 3100 m/sec, ρ = 7850 kg/m^3. The conformity between numerical and experimental results is, according to the Figs. 8 and 9 very good, the relative error does not exceed 7% in case of all measurements for both modes and both types of velocities.

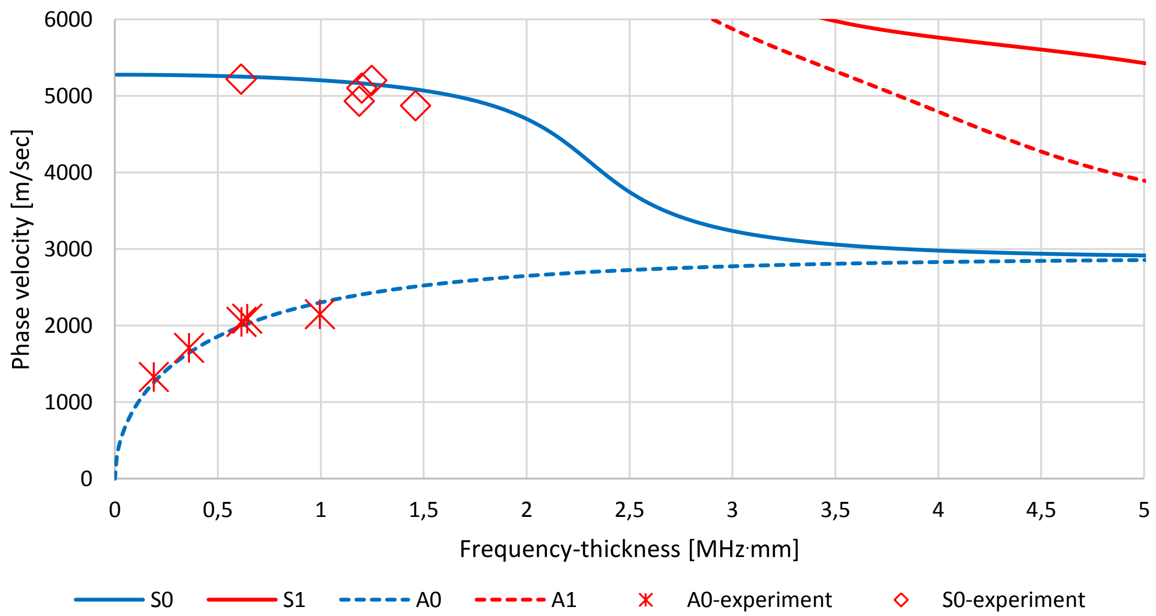

Fig. 8: Dispersion curves for selected symmetric and antisymmetric Lamb modes

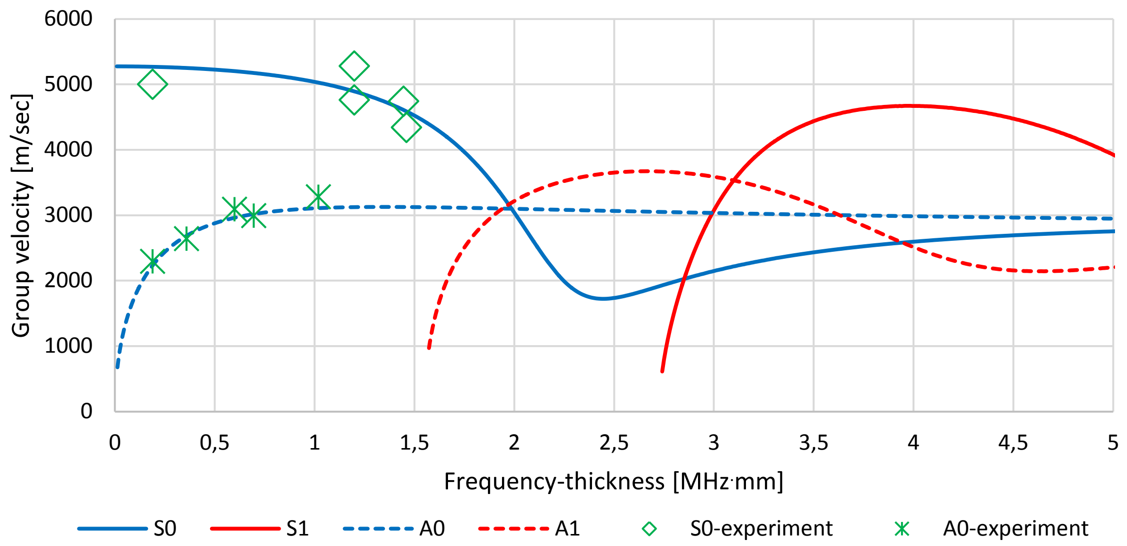

Fig. 9: Dispersion curves for selected symmetric and antisymmetric Lamb modes

Presented results show, however, better conformity for A0 Lamb mode. The reason lies in much higher displacement amplitudes in *z* axis compared to S0 mode, which will clearly reflect in the signal obtained from the transducer.

The authors have also focused on dependency of pulse central frequency and wavelength on sheet metal thickness as show Figs. 10-11. It follows, that the A0 mode retains its central frequency (Fig. 10) between 150 and 200 kHz with decreasing character when increasing the plate thickness.

The same trend can be observed for S0 mode (Fig. 10) with the difference of much broader frequency range (614 - 200 kHz versus 200 - 150 kHz in case of A0 mode). This finding is in accordance with the dependence of wavelength on the plate thickness, because of inverse relationship between wavelength and frequency. The origin of these findings is mainly in the wave structure, which fundamentally affects its propagation properties.

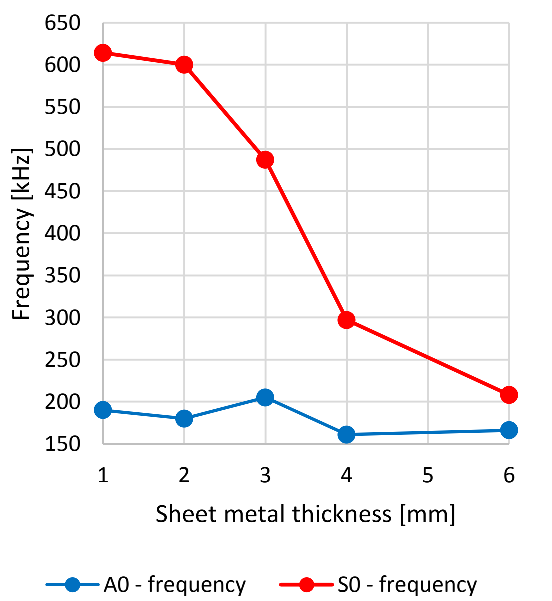

Fig. 10: Dependency of measured pulse central frequency on sheet metal thickness for A0/S0 Lamb wave modes

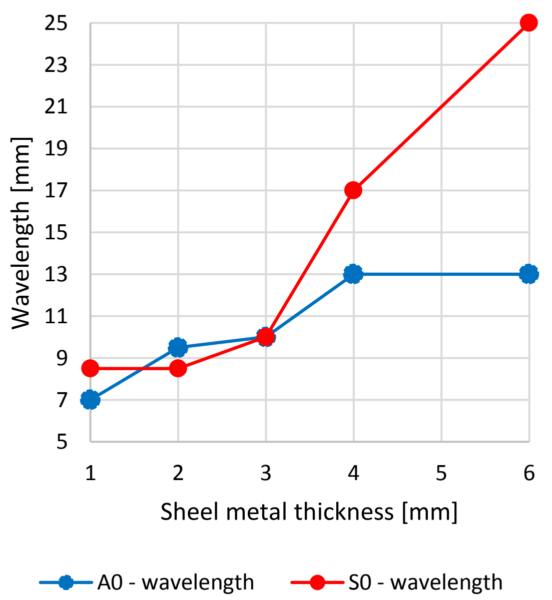

Fig. 11: Dependency of measured wavelength on sheet metal thickness for A0/S0 Lamb wave modes

Conclusions

Despite the relatively challenging evaluation of phase and group velocity of the S0 Lamb mode, the authors have achieved very good conformity between the experimental and numerical results in terms of finding dispersion curves for phase and group velocities of A0 and S0 Lamb mode. Thanks to the presence of modes with higher central frequencies, which were present during the measurement on sheet metals with thickness of 3 and 4 millimetres, it was possible to virtually extent the range of frequency-thickness product up to the value of nearly 1,5.

For future work, authors would like to focus on the obtaining of the dispersion curves for higher values of frequency-thickness product as well as deeper analyse the wave structure and its influence on wave propagation properties.

Acknowledgement

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