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Based reliability Assessment Method Applied to the Model of Structural Element

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Abstract. The article deals with possibility of application of the simulation based reliability assessment SBRA Method in modelling of structures in the field of judgement of their serviceability. A steel simply supported beam of rectangular cross sectional area was taken into account. Strains of the model using electrical-resistance strain gage method were determined. Because of the same material of the model and real beam, strains and stresses are the same in both cases. At the first step a model of the beam was created and the necessary corresponding parameters were determined for real beam using modelling rules. It means that inverse procedure of modelling was used in that case. Stresses of the model and corresponding beam were determined using SBRA Method when following random variable quantities as section modulus of the beams W, Young´s modulus of elasticity E and acting force F were taken into account.

Introduction

The article deals with an ability to assess the reliability of structural element on its model by means of the simulation based reliability assessment SBRA Method. The aim is to present a possibility of judgement of probability of failure of real component on its model. A simply supported steel beam of rectangular cross section area loaded by a simple concentrated force *F* (Fig. 3) was taken into account as a model of real steel beam. That model was available. That´s why that opposite approach was chosen when for the existing model actual corresponding beam was searched using rules of modelling. Using modelling rules the bending stresses were determined in the real beam using experimentally determined strains and stresses in the model. Stresses of steel beam and its model were determined using SBRA Method too and that method was used for judgement of their probability failure. Obtained results were compared

The simulation Based Reliability Assessment Method is a probabilistic method using the Monte Carlo simulation [**1, 3**]. Substance of that method consist in repeated calculations of relatively simple equations, where variables as dimensions of the body, mechanical properties, loads, etc. can be insert. That variables can be constant or defined by histograms, respectively. Probability of failures of model and real beam were determined using Anthill software [**3**].

Modelling of engineering problems can be very often a way to solve them. It is generally based on the conditions

$$(\pi_i)_S = (\pi_i)_M\,, \quad i = 1, 2, ..., m \tag{1}$$

where π_i are so called dimensionless parameters for structure (subscript S) and model (subscript *M*)

$$\pi = x_1^{e_1}\; x_2^{e_2}\; .....x_n^{e}\,, \tag{2}$$

if the solved problem depends on n variables x_i (expressing physical, geometrical etc. quantities). Exponents e_i, $i = 1, 2, .., n$ has to fulfil the condition of dimensionless of π terms, see [2].

$m = n - r$ represents number of independent π terms, *r* is rank of so called dimensional matrix, see [2]. There is assumed that solved problem is described by *n* physical quantities x_i, $i = 1, 2, .., n$ containing k so called primary quantities with primary units $\lfloor L_j \rfloor$, $j = 1, 2, .., k$. The necessary procedure is described in [2].

Variables incoming in the solved problem are: $l\,[mm]$ length , $W\,[mm^3]$ section modulus, $E\,[MPa]$ modulus of elasticity, $F\,[N]$ acting force , $\sigma\,[MPa]$ acting stress, ε strain. There are two primary quantities length and force with two primary units in the solved problem $[mm]$ and $[N]$. It is possible in modelling of static tasks to consider Newton as basic dimension because time and mass are not in this type of tasks separate. Then number of independent terms will be $m = 6 - 2 = 4$m. Because the strain is dimensionless quantity there are only three remaining independent terms:

$$\pi_1 = l^3 / W \tag{3}$$

$$\pi_2 = F / E\,l^2 \tag{4}$$

$$\pi_3 = \sigma / E \tag{5}$$

From Eq. 1 and Eq. 5 is obvious that stresses in the model and in the beam are for the equal material of the same magnitude. Scale for the model was chosen equal to 5. That is why the length of the beam was $l_B = 2500\,mm$.

From Eq. 1 and Eq.3 is corresponding section modulus of the beam

$$W_B = W_M \cdot l_B^3 / l_M^3 = 45.375 \cdot 10^3\ mm^3$$

and from Eq.1 and Eq. 4 is corresponding force acting on the beam

$$F_B = F_M \cdot l_B^2 / l_M^2 = 25 F_M$$

Probability of failure is guided by so called safety function

$$P_{f(i)} = R_{(i)} = S_{(i)} \ , \tag{6}$$

where **S** is the load effect And **R** is the structural resistance. The probability of failure P_f of a body can be then expressed as a ratio between the number N_f of results that do not fulfil the defined before safety function and N_t is total number of results [1], see Fig. 2.

$$P_f = \mathrm{N_f} / N_t \ . \tag{7}$$

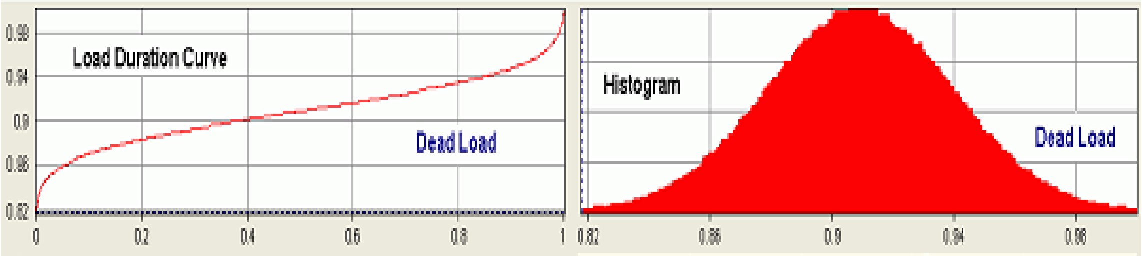

Fig. 1. Histogram of the Dead Load

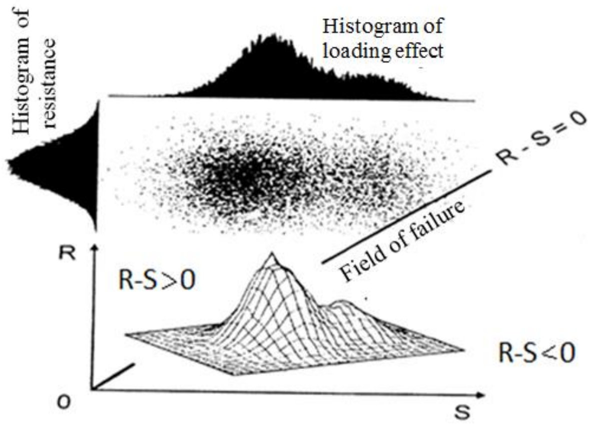

Fig. 2. Probability of failure

Experimental results

The used steel model is stated in Fig. 3. Loading forces were F = 100, 200, 300, 400, 450 N. Strains were measured using electrical-resistance gages. Their positions are obvious in Fig. 3. Recorded values of strains and corresponding stresses are in the Tab. 1.

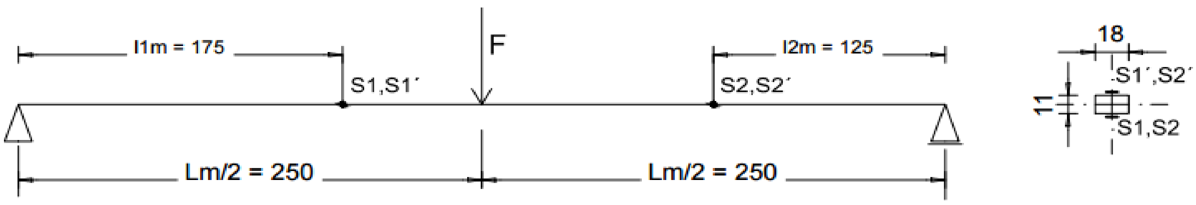

Fig. 3. Model of the beam

To determine corresponding state in real steel beam length of $l = 2500\,mm$ rules of modeling were used and loading forces F_B and section modulus W_B were determined. Material of model and of real beam was the same therefore stresses in the model and in the real beam are the same, steel S235 with Young´s modulus of elasticity $E = 2.1 \cdot 10^5$ MPa was used, see the Tab. 1.

Tab. 1 Results of experiment.

Model experiment	**F_m[N]**	**100**	**200**	**300**	**400**	**450**
	$2\varepsilon_{1m} \cdot 10^{-3}$	0.230	0.460	0.685	0.920	1.030
	σ_{1m} [Mpa]	24.15	48.30	72.45	96.60	108.15
	$2\,\varepsilon_{2m} \cdot 10^{-3}$	0.160	0.320	0.474	0.636	0.710
	σ_{2m} [Mpa]	16.80	33.60	49.77	67.78	74.55
Beam	**F_B**[kN]	2.5	5.0	7.5	10.0	11.25

Tab. 2 Results obtained using Model and SBRA simulation.

F_m[N]	**100**			**300**			**450**		
F_B[kN]	**2,5**			**7,5**			**11,25**		
	MODEL exp.	**SBRA-m**	**SBRA-B**	**MODEL exp.**	**SBRA-m**	**SBRA-B**	**MODEL exp.**	**SBRA-m**	**SBRA-B**
$2\varepsilon_{1m} \cdot 10^{-3}$	0.230	-	-	0.685	-	-	1.030	-	-
$2\varepsilon_{2m} \cdot 10^{-3}$	0.160	-	-	0.474	-	-	0.710	-	-
σ_1 [MPa]	24.15	24.1089	24,1131	72.45	72.3371	72.3305	108.15	108.5044	108.5013
σ_2 [MPa]	16.80	16.8109	16.8094	49.77	50.4316	50.4297	74.55	75.6430	75.6484
$\sigma_{1Det.}$[MPa]	24.104			72.314			108.468		
$\sigma_{2Det.}$[MPa]	16.804			50.413			75.618		

m - Model, B – Beam, Det. – beam determined

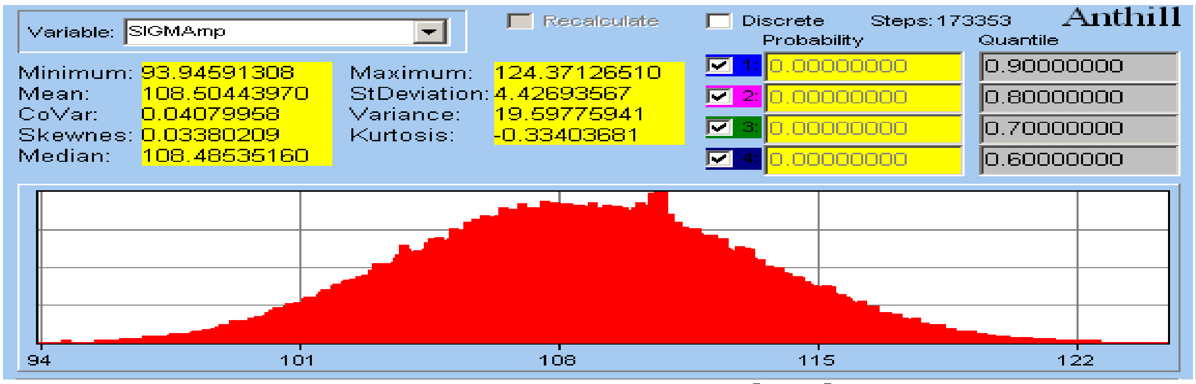

Fig. 4. The resulting stress $\sigma_{lm} = [MPa]\,\sigma_{1m}$.

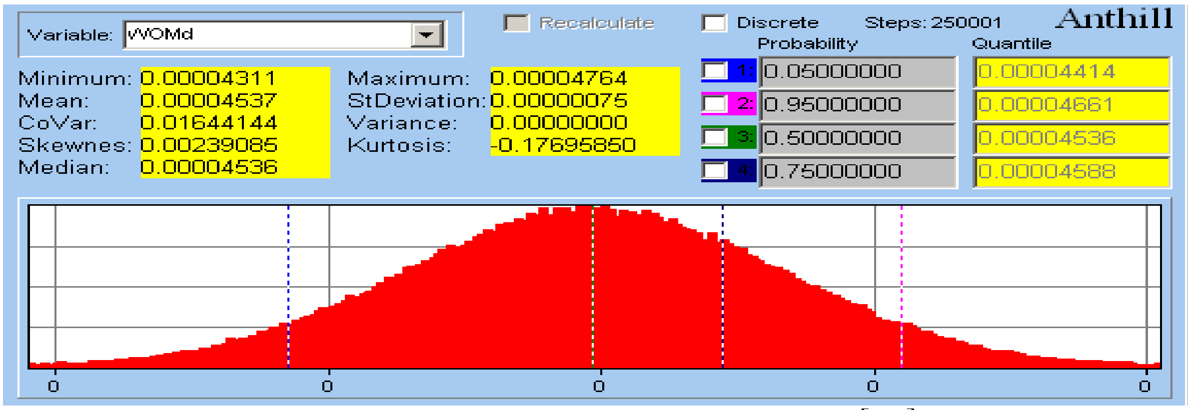

Fig 5. The histogram section modulus $\mathbf{W}_B = \left[m^3\right]$.

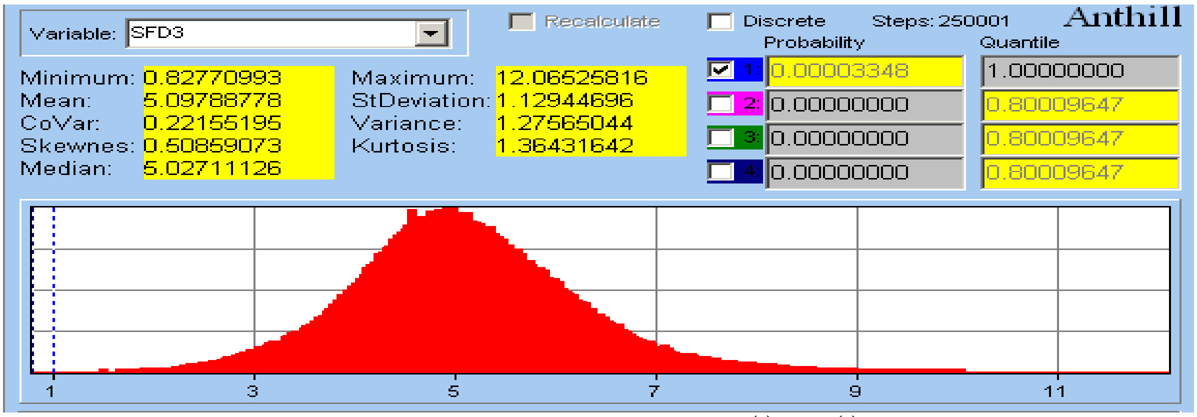

Fig. 6. Probability of failure $Pf(i)$, $SF(i)$.

In Fig.4. and Fig.5.there are for illustration presented histograms of the resultant stress and section modulus and in Fig. 6. histogram of probability of failure.

Probability of failures of the beam determined using SBRA method were probabilities of failure $\mathbf{P}_{f(i)B} = 3.476 \cdot 10^{-5}$ for beam and $\mathbf{P}_{f(i)m} = 3.348 \cdot 10^{-5}$ for the model

Conclusions

Obtained results for the model and for the beam show very good correspondence. It gives a sure possibility for determination of failure probability of structure to determine it using corresponding model.

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