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Modelling of the flow of streams of cohesionless and cohesive bulk materials in a conveyor discharge point with a flat conveyor belt

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Abstract. The paper presents the analysis of flow conditions of cohesive and cohesionless bulk materials in a conveyor discharge point of a flat conveyor belt. The analysis was carried out for stationary flows at high velocities. It presents mathematical methods of description of velocity of material leaving a throwing point of a flat conveyor belt as well as final equations which enable the determination of velocity of the material after it has left the throwing point (with the accuracy sufficient for practical use). Next, the velocity calculated for the proposed mathematical description (for selected material groups) has been compared with the velocity obtained from mathematical relations commonly used by engineers. The proposed equations for determining the velocity of material beyond the point have been proved useful, since they enable excluding the indirect equations. Finally, the difference between the values of velocity obtained with the proposed and indirect equations has been determined and the relative error for the proposed method has been calculated.

1. Introduction

In transport conveyor systems built on the basis of belt conveyors, pouring points are important parts. They are crucial elements allowing transporting bulk materials on different levels of this type of system. An important issue in a conveyor transport is to ensure proper flow of material through pouring points. This involves ensuring continuity of the transported material stream and ensuring the required transport capacity of the system. Pouring points used in transport systems often take advantage of gravity to transport materials. These types of pouring points include e.g. chutes [1], [20]. Another pouring points that cooperate with belt conveyors are impact plates [7], [9], [10], [11], [16], [18], [19]. They are to slow down the velocity of material to the value corresponding with the velocity of the receiving conveyor in the feed point.

A throwing point with a flat conveyor belt, discussed further, uses a driven belt for moving and throwing the material. However, the point is not inclined as described in [5], [6], [8], [14]. This means that the angle of inclination of the conveyor belt is 0° [16], [17].

In transport systems, slow belt conveyors are most commonly used. They operate with velocities below 3 m/s [1], [20], and equations determining the velocity of cohesive and cohesionless material beyond the conveyor pulley are described in [14]. However, these equations should not be used for fast conveyors. Equations for fast conveyors are presented in [14]. They do not take adhesion into consideration, but include the influence of air resistance on the velocity of the transported material beyond the head pulley. In the work [17], the influence of adhesion on the velocity of material beyond the point was also omitted. However the work included material, kinematic and dynamic parameters. In the work [4] the adhesion phenomenon also was omitted.

In the case of discharge belt conveyors [15], the construction of this type of conveyors and the discharge method of the material are different from the ones used for a belt conveyor feeding the next pouring point. Thus, equations proposed in [15] designed for estimation the velocity of material beyond the pulley, should not be used for fast feeding belt conveyors.

In [12], only kinematic and geometric parameters for estimation of the velocity of material falling down onto a belt conveyor were taken into consideration, whereas in [2], authors discuss simple engineering equations describing the velocity of material flowing from a belt conveyor. They take into consideration the coefficient of friction of the material against the belt but do not take into consideration forces and adhesion affecting onto the material stream.

Works [5], [6], [8] take into account kinematic, dynamic and material parameters including adhesion, but equations presented there are designed for inclined conveyors with ascending and/or descending belts.

Author in [7] analyse the trajectory of material beyond a throwing point depending on the velocity of a belt conveyor, but they do not analyse the velocity of the material beyond the point where it reaches the next pouring point. In addition, work [3] refers to the prediction of the trajectory of the material beyond the throwing point, but in this case, both fast and slow conveyors were taken into consideration.

Therefore, it was important to propose equations describing the velocity of material beyond the pulley of a not inclined belt conveyor, equipped with the belt without cleats for both cohesive and cohesionless materials.

2. Analysis of flow conditions of cohesionless and cohesive materials in the throwing point with a flat belt

In conveyor transport systems, a throwing point refers to a head pulley of the belt conveyor. From the pulley, the material is transferred further, as a feed, onto another conveyor, which, most often, is located lower and the feed is delivered with the use of a chute [1], [20] or an impact plate [1], [6], [7], [9-11], [16], [20]. The material from the throwing point may also be thrown directly onto hillocks.

The analysis of flow conditions of cohesive and cohesionless bulk materials in a throwing point with a flat belt is shown in Figure 1 and 2. The analysis was carried out both for fast conveyors, where the belt transports the material with the velocity above 3 [m/s], and for stationary flow. A belt without cleats was analysed and the influence of air resistance was not considered.

Figures 1 and 2 illustrate magnitudes taken into consideration in the analysis of the flow conditions that affect the process of discharging the material from the conveyor’s pulley. Geometric, kinematic and dynamic conditions of the motion of elementary mass dm in a throwing point, which is not inclined, were taken into consideration.

In a throwing point discharging cohesionless materials, the elementary mass dm is subject to the following forces describing dynamic conditions of its flow (Fig. 1): gravitational force dG [N], centrifugal force dF [N], normal force dN_{xs} [N], tangential force dT_{xs} [N]. While determining the relations describing the discharge of the martial from the pulley, the inertial force dJ [N] formed as a result of action of gravitational force dG, centrifugal force dF and forces dN_{xs}, dT_{xs} was also considered.

In the case of cohesive materials, the dynamic conditions of the flow of the material in the throwing point, besides the above mentioned forces, also adhesion force dF_a (Fig. 2) was considered. The adhesion force forms between the belt and the transported material.

Kinematic conditions of the flow of material in a throwing point with a flat belt (Fig. 1 and 2) include: the velocity of the conveyor belt v_t [m/s], the velocity of bulk material stream flowing into the head pulley v_p [m/s], which is equal to the velocity of the belt, and the velocity of material leaving the head pulley v_w [m/s].

Geometric conditions of material flowing out from the throwing point include the following parameters [8]: angle of conveyor descent α [°] (in this case α=0°), angle of the material stream flowing out of the head pulley β_t [°] (in this case β_t is the angle of material flowing into another point, e.g. impact point), angle coordinate which describes the position of infinitesimal mass of material on the head pulley φ [°] [8], radius of the head pulley r_b [m], the average radius of curvature of material stream $R_{śr}$ [m], thickness of the layer of material stream at the moment it flows onto the head pulley h_m [m] [8], thickness of the layer of material stream leaving the head pulley h_w [m] [8].

Kinematic and geometric conditions of material flowing out of a head pulley with a flat belt for cohesive and cohesionless materials are the same. whereas in the case of dynamic conditions – the difference is the adhesion force appears in case of cohesive materials.

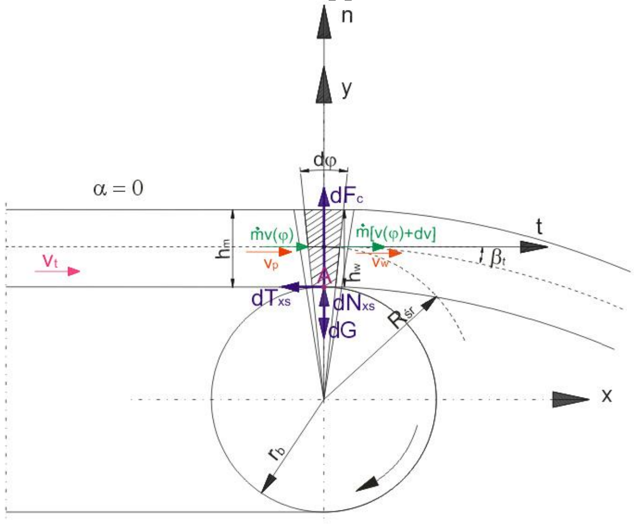

Fig. 1. Geometric, kinematic and dynamic conditions of the flow of a stream of cohesionless bulk material on a head pulley of a conveyor with a flat belt [own elaboration]

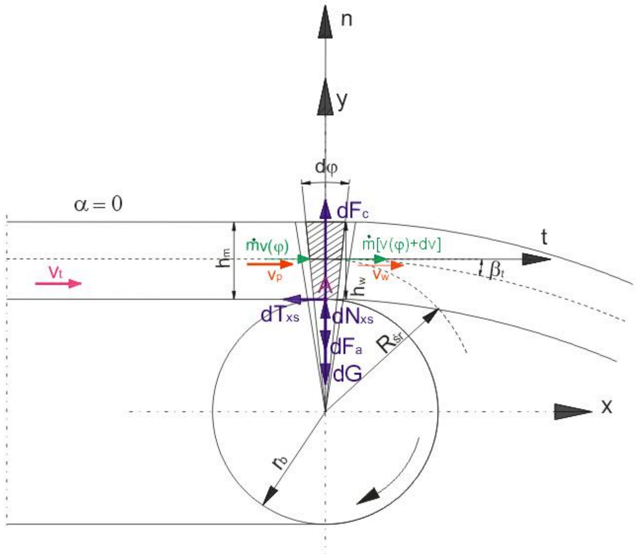

Fig. 2. Geometric, kinematic and dynamic conditions of the flow of a stream of cohesive bulk material on a head pulley of a conveyor with a flat belt [7]

3. Mathematical model of the flow of a stream of cohesionless and cohesive bulk material in a throwing point with a flat belt

In the analysis of the discharge of material from a throwing point with a flat belt, it was assumed that the material is separated from the pulley in point A (Fig. 1 and 2). This is the point in which the belt overlaps the head pulley. Regardless of the type of transported material, Equation 1 must be satisfied to let the material leave the belt [1]:

$$\frac{v^2}{R} > g \cdot \cos\alpha \tag{1}$$

where: v – belt velocity [m/s], R - radius of the pulley [m], g – gravitational acceleration [m/s^2], α - the angle of the conveyor descend [°] [1], [8].

For cohesive materials, also Equation 2, taking into consideration adhesion and high velocities of the conveyor, must be satisfied [14]:

$$\frac{F_r - A_d}{\cos\alpha} \geq 1 \tag{2}$$

where:

$$A_d = \frac{\sigma_a}{\gamma \cdot h_m} \tag{3}$$

$$F_r = \frac{v^2}{g \cdot R} \tag{4}$$

where: v – belt velocity [m/s], γ - specific gravity of bulk material [N/m3], σ_a – adhesion [N/m^2], h_m - thickness of the material stream [m], g – gravitational acceleration [m/s^2], R - radius of the pulley [m] [8].

To obtain mathematical relations allowing determination of velocity v(φ) [m/s] of cohesive and cohesionless bulk material beyond the throwing points with a flat belt, a system of equations was used. The system differs depending on the type of material, and for cohesionless material it includes:

- equation of continuity [8]:

$$\dot{m} = \rho \cdot v(\varphi) \cdot A(\varphi) \tag{5}$$

- equation of equilibrium [8]:

$$\dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + d\vec{N}_{xs} + d\vec{T}_{xs} \tag{6}$$

- equation describing the contact friction condition on surfaces of walls restricting the path of the flow of a bulk material stream for static conditions and cohesionless materials (according to [14]):

$$\tau_{ws} = \sigma_n \mu_{xs} \tag{7}$$

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- equation describing the contact friction condition on the surfaces of walls restricting the path of the flow of a bulk material stream for static conditions and cohesive materials (according to [14]):

$$\tau_{ws} = (\sigma_n + \sigma_a)\mu_{xs} \tag{10}$$

In Equations 7 and 9, μ_{xs} is a contact friction coefficient for static conditions. The general form of the system of equations for cohesionless material takes the form 11 [8]:

$$\begin{cases} \dot{m} = \rho \cdot v(\varphi) \cdot A(\varphi) \\ \dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + d\vec{N}_{xs} + d\vec{T}_{xs} \\ \tau_{ws} = \sigma_n \mu_{xs} \end{cases} \tag{11}$$

whereas for cohesive materials, the form of the system of equations is as follows [8]:

$$\begin{cases} \dot{m} = \rho \cdot v(\varphi) \cdot A(\varphi) \\ \dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + \vec{F}_a + \left|d\vec{N}_{xs}\right| + \left|d\vec{T}_{xs}\right| \\ \tau_{ws} = (\sigma_n + \sigma_a)\mu_{xs} \end{cases} \tag{12}$$

A detailed description of parameters included in the systems of Equations 11 and 12 is the same as presented in publication [8], where $A(\varphi)$ – cross-section of the stream [m^2], ρ - bulk material density [kg/m^3], $\dot{m}$ - mass flow [kg/s].

Projection of Equation 6 onto directions of the assumed coordinate system <n,t> (Fig. 1) will generate a system of equation as a function of angular parameter φ, which after simplification, for cohesionless materials, takes the form:

$$\begin{cases} \mu_{xs}\left|d\vec{F}_c\right| - \mu_{xs}\left|d\vec{G}\right| = 0 \\ \dot{m}\, dv = 0 \end{cases} \tag{13}$$

For cohesive materials projection of Equation 9 onto direction of the assumed coordinate system <n,t> (Fig. 2) will also generate a system of equations as a function of angular parameter φ, a simplified form of which is as follow:

$$\begin{cases} \mu_{xs}\left|d\vec{F}_c\right| - \mu_{xs}\left|d\vec{F}_a\right| - \mu_{xs}\left|d\vec{G}\right| = 0 \\ \dot{m}\, dv + \mu_{xs}\left|d\vec{F}_a\right| = 0 \end{cases} \tag{14}$$

The solutions to Equations 13 and 14 are respectively differential equations, where for cohesionless materials the solution takes the following form 15:

$$\frac{dv^2(\varphi)}{d\varphi} - 2\,\mu_{xs}\, v^2(\varphi) = -2\, g\, \mu_{xs} R(\varphi) \tag{15}$$

and for cohesive materials – the solution is presented by 16:

$$\frac{dv^2(\varphi)}{d\varphi} - 2\,\mu_{xs}\, v^2(\varphi) = -2\,\mu_{xs}\, g\, R(\varphi)\left[1 + \frac{2\sigma_a}{\gamma h(\varphi)}\right] \tag{16}$$

Equations 15 and 16 are a special case of the Bernoulli equation, which can be written in the form (according to [16]):

$$y' + P(\varphi)y = Q(\varphi) \tag{17}$$

By integrating Equation 17, equations that allow determining the velocity of the material beyond the throwing point for cohesive and cohesionless materials were obtained. For cohesionless materials the velocity is described by Equation 18:

$$v_w = \sqrt{C_w\, e^{2\mu_{xs}\varphi} + g\, R(\varphi)} \tag{18}$$

and for cohesive materials by Equation 19:

$$v_w = \sqrt{C_w\, e^{2\mu_{xs}\varphi} + g\, R(\varphi)\left[1 + \frac{2\sigma_a}{\gamma h(\varphi)}\right]} \tag{19}$$

The description of parameters included in Equations 11 and 12 is the same as presented in publication [8], where C_w - integration constant, R(φ) - an average radius of curvature of the stream [m], h φ) - thickness of the material stream (according to Fig. 2, it is the height of the material stream lying on the conveyor belt).

Equations 18 and 19 contain integration constants, which describe Equation 20 for cohesionless materials, and Equation 21 for cohesive materials:

$$C_W = e^{-2\mu_{xs}\varphi}\left\{v_p^2 - g\,R(\varphi)\right\} \tag{20}$$

$$C_W = e^{-2\mu_{xs}\varphi}\left\{v_p^2 - g\,R(\varphi)\left[1 + \frac{2\sigma_a}{\gamma h(\varphi)}\right]\right\} \tag{21}$$

Equations 18 and 19 as well as 20 and 21 differ one from another by an element consisting adhesion.

To obtain a solution, boundary conditions, which are the same for both cohesive and cohesionless materials, were taken. Thus, the following boundary conditions, allowing determining the integration constant, were assumed [5], [6]:

$$\varphi = \alpha \tag{22}$$

$$R(\varphi) = R(\alpha) = R_0 = r_b + 0{,}5h_m \tag{23}$$

$$h_m = \frac{\dot{m}}{\rho v_t B} \tag{24}$$

$$v(\varphi) = v(\alpha) = v_0 = v_t \tag{25}$$

$$A(\varphi) = A(\alpha) = A_0 = \frac{\dot{m}}{v(\alpha)\rho} = \frac{\dot{m}}{v_t\,\rho} = h_m\,B \tag{26}$$

Velocity v_w allows assuming the following boundary condition [5], [6]:

$$\varphi = \varphi_W = \beta_t \tag{27}$$

The value of the velocity of material beyond the throwing point is obtained by applying a simple iteration method, assuming initial conditions determined from Equations 22-27.

To obtain the exact approximation of velocity v_w of material leaving the point, the following relation should be satisfied:

$$\left|\frac{v_{wn} - v_{w(n-1)}}{v_{wn}}\right| \le \delta_{v_W}\ [\%] \tag{28}$$

where δ_{vw} is an acceptable relative deviation of the estimated velocity v_w. It is assumed that velocity v_w was correctly determined if the value of the deviation ranges 1%-2%.

Parameter α is constant because the conveyor is not inclined, unlike in solutions presented in [5], [6], and [8].

Equations 18 and 19 were tested to verify if they comply with the assumption that the velocity of material directly leaving the head pulley of the conveyor is equal to the velocity of the belt. Both equations proved to be compliant with the assumption.

Equations 18 and 19 can also be used to determine the velocity of material beyond the throwing point, in the function of depending on the assumed angle of material inflow to the next point, e.g. an impact plate (Fig. 3). In this case, it may be assumed in the proposed equations that angle β_t is equal to the angle at which the material falls onto a surface (e.g. impact plate). Such an approach is a simplification, but returns results sufficient for practical use.

4. Indirect equations

Indirect equations allow determining the velocity of material leaving the throwing point which cooperates with an impact point presented in [16], where they were applied for a conveyor with an ascending and descending belt. These equations can also be used for a flat conveyor. They are suitable mainly for a conveyor supplied with an impact plate. They return best results for such application.

Figure 3 presents the flow conditions of material on a conveyor with a flat belt (the conveyor is not inclined), whereas Equations 29-32 allow determining the numerical values of parameters that can be seen in Figure 3. These equations are usable regardless the material.

Indirect equations (Fig. 3) include [16]:

- equation describing an angle of material flowing onto impact point ξ_0:

$$\xi_0 = \operatorname{arctg}\left(tg\beta_t - \frac{g \cdot x_0}{2 \cdot v_{w1}^2 \cdot \cos^2 \beta_t} \right) \tag{29}$$

- equation describing the velocity of material flowing onto impact point v_0:

$$v_0 = v_{w1} \cdot \cos\beta_t \cdot \sqrt{tg^2 \xi_0 + 1} \tag{30}$$

- equations describing distance parameters, according to Fig. 3:

$$x_0 = s_0 + (r_b + 0{,}5 \cdot h_0) \cdot \sin\beta_t \tag{31}$$

$$y_0 = tg\beta_t \cdot x_0 - \frac{g \cdot x_0^2}{2 \cdot v_{w1} \cdot \cos^2 \beta_t} \tag{32}$$

where β_t is an inclination angle of the material stream leaving the conveyor. In this case β_t is equal 0 because the conveyor is not inclined.

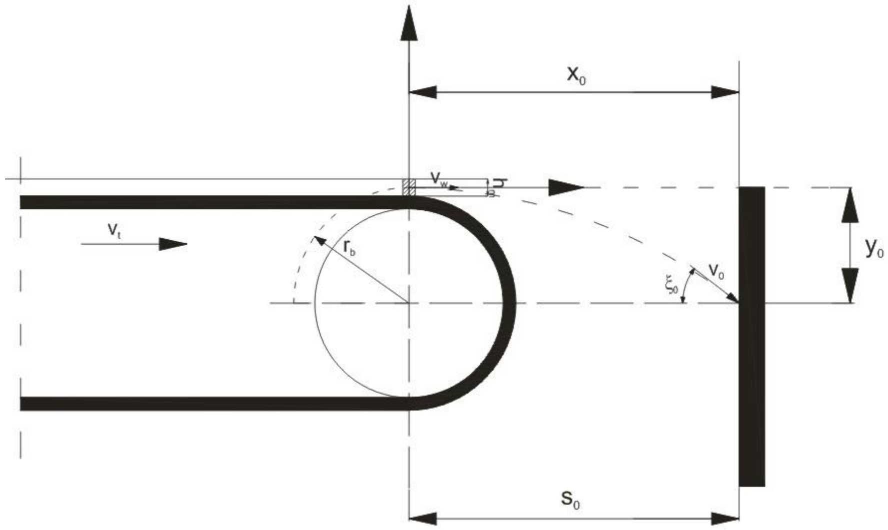

Fig.3. Cooperation of a throwing point with a flat belt and an impact point with a not inclined plate [elaborated based on [16]]

5. Comparison of the presented calculation methods

Table 1 was presents an example demonstrating the usefulness of the proposed equations. It also includes a comparison between the results obtained with the use of the proposed equations and the results obtained with the currently used indirect equations. In the proposed equations, the indirect equations are omitted, assuming that the angle of the outflow of material from a head pulley is equal to the angle of the inflow of material to the impact plate ($\beta_t=\xi_0$). This is a considerable simplification in relation to the currently used indirect equations, where not only the velocity of material flowing into the impact plate, but also the angle at which it flows into the plate are determined.

Table 1 presents the comparison of the results obtained with indirect equations and with the proposed equations for both cohesive and cohesionless materials. Table 1 also contains estimation of the percentage error for results obtained from the proposed equations in relation to the results obtained with the use of indirect equations. It also presents the difference between the obtained velocities of material at the moment when it falls onto the plate. Initial conditions of the material outflow are the same for both calculation cases.

Some of the values describing the parameters of the outflow of material from the throwing point were taken from [8] and [20].

Table 1. Determination of the velocity of material beyond the discharge point in relation of the analyzed angle at which material flows into impact point [own elaboration]

Cohesionless material	Cohesive material
- required mass flow $\dot m$ = 350 [kg/s], - belt velocity v_p = 3,15 [m/s], - belt width B_t = 0,8 [m], - pulley radius r_b = 0,25 [m], - angle of inclination of the conveyor belt α = 0 [^{o}], - gravitational acceleration g = 9,81 [m/s^2], - material bulk density ρ = 850 [kg/m^3], - specific gravity γ = 8340 [N/m^3], - friction coefficient for static conditions μ_{xs} = 0,50, - angle at which bulk material flows into subsequent point β_t= ξ0 = 41 [^{o}], - distance between the head pulley and the point at which material is discharged s_0= 0,9 [m],	- required mass flow $\dot m$ = 350 [kg/s], - belt velocity v_p = 3,15 [m/s], - belt width B_t = 0,8 [m], - pulley radius r_b = 0,25 [m], - angle of inclination of the conveyor belt α = 0 [^{o}], - gravitational acceleration g = 9,81 [m/s^2], - material bulk density ρ = 1380 [kg/m^3], - specific gravity γ = 13540 [N/m^3], - friction coefficient for static conditions μ_{xs} = 0,51, - angle at which bulk material flows into subsequent point β_t= ξ0 = 41 [^{o}], - distance between the head pulley and the point at which material is discharged s_0= 0,9 [m], - adhesion σ_a = 350 [N/m^2],
CALCULATIONS - *proposed equations* the first step of approximation boundary conditions determining the integration constant $\varphi = \alpha = 0$ $v(\varphi)=v(\alpha)=v_0=v_t=3{,}1$ [illegible] $h_m=\frac{A_v}{B_t}=\frac{\dot m}{B_t\rho v_t}=\frac{350}{[illegible]}=$ [illegible] $A_0=A_v=\frac{\dot m}{v_t\rho}=h_m B_t=$ [illegible] $R_{w1}=$ [illegible] integration constant $C_{w1}=e^{-2\mu_{xs}\alpha}\left\{v_t^2-g\,R_{w1}\right\}$ $C_{w1}=6{,}671$ boundary condition determining the velocity $\varphi=\varphi_w=\beta_t=4$ [illegible] velocity	CALCULATIONS - *proposed equations* the first step of approximation boundary conditions determining the integration constant $\varphi = \alpha = 0$ $v(\varphi)=v(\alpha)=v_0=v_t=3{,}1$ [illegible] $h_m=\frac{A_v}{B_t}=\frac{\dot m}{B_t\rho v_t}=\frac{350}{[illegible]}=$ [illegible] $A_0=A_v=\frac{\dot m}{v_t\rho}=h_m B_t=$ [illegible] $R_{w1}=$ [illegible] integration constant $C_{w1}=e^{-2\mu_{xs}\alpha}\left\{v_t^2-gR_{w1}\left[1+\frac{2\sigma_a}{\gamma h_m}\right]\right\}$ $C_{w1}=6{,}976$ boundary condition determining the velocity $\varphi=\varphi_w=\beta_t=4$ [illegible] velocity

$v_{w1}=\sqrt{C_{w1}e^{2\mu\beta_s}-gR_{w1}}$ $v_{w1} = 4{,}110$ deviation value $\left\lvert\frac{v_{wn}-v_{w(n-1)}}{v_{wn}}\right\rvert\cdot 100\% \le \varepsilon_v$ $\delta_{v_{w1}} = 2{,}36 > 2\%$ approximation is insufficient the second step of approximation $h_{w1}=\frac{A_{w2}}{B}=\frac{\dot{m}}{\rho v_{w1}B}=\frac{350}{[illegible]}=[illegible]$ $A_{w2}=\frac{\dot{m}}{v\rho}=h_{w1}B=[illegible]$ $R_{w2}=R_{s}+\frac{?}{?}h_{w1}=[illegible]$ integration constant $C_{w2}=e^{-2\mu\alpha_s}\left(v_t^2-gR_{w2}\right)$ $C_{w2} = 6{,}855$ velocity $v_{w2}=\sqrt{C_{w2}e^{2\mu\beta_s}-gR_{w2}}$ $v_{w2} = 4{,}133$ deviation value $\delta_{v_{w2}} = 0{,}57 < 2\%$ - *indirect equations* $\beta_t = \alpha = 0$ $x_0 = s_0 + \left(R_{w1} - 0{,}5h_m\right)\sin\beta_t$ $x_0 = 0{,}9$ $\xi_0 = \operatorname{arctg}\left(\operatorname{tg}\beta_t + \frac{gx_0}{2v_{w1}^2\cos^2\beta_t}\right)$ $\xi_0 = 41$ $v_0 = v_t\cdot\cos\beta_t\sqrt{\operatorname{tg}^2\xi_0 + 1}$ $v_0 = 4{,}216$ - relative error of the determination of the velocity with both methods $\delta_{v_r} = 2\%$	$v_{w1}=\sqrt{C_{w1}e^{2\mu\beta_s}-gR_{w1}\left[1+\frac{2\tau_a}{\gamma h_m}\right]}$ $v_{w1} = 4{,}351$ deviation value $\left\lvert\frac{v_{wn}-v_{w(n-1)}}{v_{wn}}\right\rvert\cdot 100\% \le \varepsilon_v$ $\delta_{v_{w1}} = 2{,}769 > 2\%$ approximation is insufficient the second step of approximation $h_{w1}=\frac{A_{w2}}{B}=\frac{\dot{m}}{\rho v_{w1}B}=\frac{350}{[illegible]}=[illegible]$ $A_{w2}=\frac{\dot{m}}{v\rho}=h_{w1}B=[illegible]$ $R_{w2}=R_{s}+\frac{?}{?}h_{w1}=[illegible]$ integration constant $C_{w2}=e^{-2\mu\alpha_s}\left\{v_p^2-gR_{w2}\left[1+\frac{2\tau_a}{\gamma h_{w1}}\right]\right\}$ $C_{w2} = 7{,}112$ velocity $v_{w2}=\sqrt{C_{w2}e^{2\mu\beta_s}-gR_{w2}\left[1+\frac{2\tau_a}{\gamma h_{w1}}\right]}$ $v_{w2} = 4{,}422$ deviation value $\delta_{v_{w2}} = 1{,}61 < 2\%$ - *indirect equations* $\beta_t = \alpha = 0$ $x_0 = s_0 + \left(R_{w1} - 0{,}5h_m\right)\sin\beta_t$ $x_0 = 0{,}9$ $\xi_0 = \operatorname{arctg}\left(\operatorname{tg}\beta_t + \frac{gx_0}{2v_{w1}^2\cos^2\beta_t}\right)$ $\xi_0 = 41$ $v_0 = v_t\cdot\cos\beta_t\sqrt{\operatorname{tg}^2\xi_0 + 1}$ $v_0 = 4{,}216$ - relative error of the determination of the velocity with both methods $\delta_{v_r} = 5\%$

The relative error δ_{vr} for results achieved with the proposed equations and with the indirect equations amounts respectively 2% for cohesionless materials and 5% for cohesive materials, which is acceptable for practical use. But the value of the error increases with the growing distance between the impact plate and head pulley. The difference between the velocity

determined with the use of the indirect equations and the proposed equations for cohesionless materials amounts 0,08 [m/s], and for cohesive materials amounts 0,21 [m/s]. The minimal permissible distance between the throwing point and the impact plate was taken into consideration.
Differences obtained in calculations for values of velocity allow statement that the proposed equations can be used for estimation of the velocity of material beyond the throwing point. The indirect equations take into consideration kinematic and geometric conditions, whereas the proposed equations take also into consideration dynamic parameters, adhesion, the coefficient of friction of material against the belt surface, as well as material parameters such as material bulk density and specific gravity.

6. Conclusions

The presented analysis of the conditions of the flow of cohesionless and cohesive materials in a throwing point with a flat belt (non inclined conveyor) as well as the proposed equations obtained from the analysis are suitable for engineering calculations and for estimation of the velocity of material beyond the head pulley with the accuracy sufficient for practical use. It is especially important if the value of the velocity beyond the point must be known, which enables the correct selection of parameters of pouring points that cooperate with conveyors, and for maintaining the constant capacity of the transport system. The proposed equations consider not only kinematics and dynamics parameters of the flow but also material parameters.

The proposed equations, comparing to the indirect equations, give good compliance of 2% for cohesionless materials and 5% for cohesive materials. Such value of the relative error was obtained for an impact plate operating in angular pouring points, whereas for parallel pouring points, where distance between the head pulley and impact plate increases to 1.2 m [20], the relative error can achieves almost 10%.
The proposed equations provide a tool for engineers allowing estimating the value of velocity of material beyond the throwing point, which assures the correct capacity of the transport system consisting of belt conveyors. The proposed equations prove to be useful in cases where a throwing point follows or precedes an impact point, especially in properly selected distance between cooperating points, but they don't consider the air resistance.
The proposed equations assume that the angle at which material falls onto the subsequent point is known. However, the equations may turn out to be useless for the assumed angle at which material flows out of the head pulley if the distance is larger than 2 m.

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