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Determining modal parameters of mechanical system by using enhanced frequency response function

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Abstract. The paper deals with determining the modal parameters of mechanical system by using Enhanced Frequency Response Function which is obtained from Complex Mode Indicator Function. The aim of the paper is to present the advantages of this approach for a case when the FRF matrix consists of many degrees of freedom. The analyzed object is a system with four degrees of freedom with viscous damping. A model of the system and all calculations are realized in Matlab. The modal parameters are determined by two different ways. In the first case, the modal parameters are determined for a spatial model by eigenvalue calculation. In the second case, the modal parameters are estimated in the frequency domain using an iterative Rational Fraction Polynomial method from enhanced frequency response functions. At the end, the computational efficiency of the proposed approach is studied.

Introduction

Modal analysis is a part of mechanics. It serves to analyze and to determine dynamic properties of structure. These features include natural frequency, mode shape and damping. Analyzed system can be described by a spatial model (mass matrix $[M]$, damping matrix $[B]$ and stiffness matrix $[K]$), modal model (spectral matrix $\left[\lambda^2\right]$ and modal matrix $[\Phi]$) and response model (frequency response function matrix $\left[H(\omega)\right]$) [1, 2]. Modal analysis can be divided into the following approaches: a) experimental modal analysis, where frequency response functions are obtained experimentally; b) numerical or analytical modal analysis, where frequency response functions are computed from spatial model [3].

Analytical solution

Let us consider four degree of freedom system (Fig. 1) with vicious damping. Its mathematical model is given by system's matrices:

$$[M]=\begin{bmatrix}2&0&0&0\\0&3&0&0\\0&0&4&0\\0&0&0&5\end{bmatrix},\quad [K]=\begin{bmatrix}3.10^5&-2.10^5&0&0\\-2.10^5&5.10^5&-3.10^5&0\\0&-3.10^5&5.10^5&-2.10^5\\0&0&-2.10^5&3.10^5\end{bmatrix},\quad [B]=\begin{bmatrix}30&-20&0&0\\-20&50&-30&0\\0&-30&50&-20\\0&0&-20&30\end{bmatrix}. \tag{1}$$

where $[M],[B],[K]$ are the mass, damping and stiffness matrices.

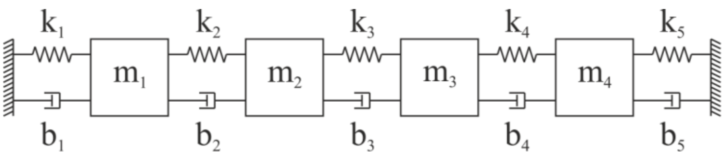

Fig. 1 Four degrees of freedom system.

The frequency response functions (FRFs) of the system was calculated analytically and are shown in Fig. 2a. All together represent the FRF matrix $\left[H(\omega)\right]$ that is used in the next calculations. Modal parameters were determined by eigenvalue solution and are listed in Table 1. Mode shapes are shown in Fig. 2b.

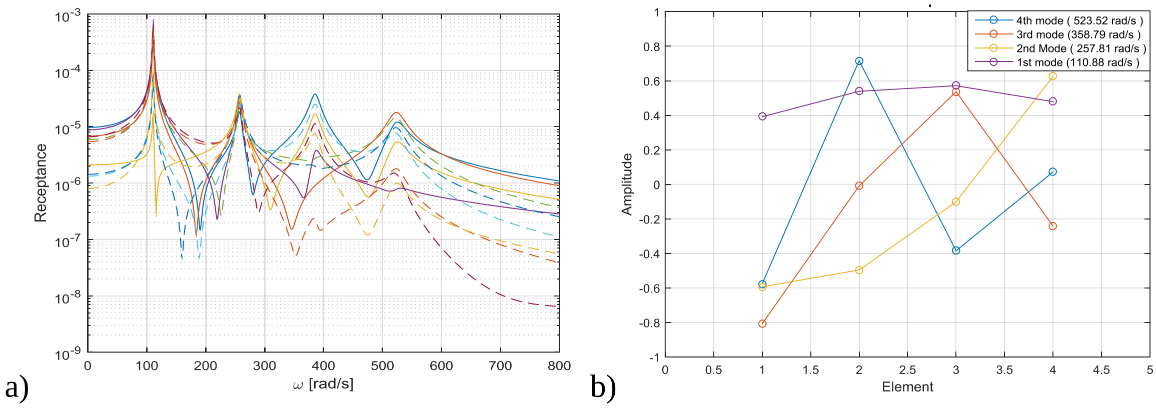

Fig. 2 a) FRFs of the system, b) Mode shapes of vibration.

Table 1 Modal parameters of the system determined analytically.

Mode	**1.**	**2.**	**3.**	**4.**
Frequency [Hz]	17.65	41.03	61.40	83.32
Damping ratio [%]	0.55	1.29	1.93	2.62

Modal parameter estimation

The complex mode indicator function (CMIF) is based on singular value decomposition of FRF matrix and can be used to initial estimate of natural frequencies of the system. Singular value decomposition can be represented by equation:

$$\left[H(\omega)\right]=\left[U(\omega)\right]\left[\Sigma(\omega)\right]\left[V(\omega)\right]^{H}, \tag{2}$$

where $\left[\Sigma(\omega)\right]$ is a diagonal matrix of singular values, $\left[U(\omega)\right]$ is a left singular matrix and $\left[V(\omega)\right]$ is a right singular matrix. Mode shapes are represented by the left singular vectors of the matrix $\left[U(\omega)\right]$. The right singular vectors in matrix $\left[V(\omega)\right]$ represent the corresponding modal participation factors. CMIF is equal to the square of the singular value magnitude: [6]

$$\mathrm{CMIF}_k(\omega)=\Sigma_k^2(\omega)\,. \tag{3}$$

Peaks detected in the CMIF plot indicate the existence of modes, and the located frequencies give the corresponding damped natural frequencies. The CMIF method is able to distinguish closed and also coupled modes.

The CMIF plot of the example problem is shown in Fig. 3. Four peaks can be seen there and every one of them indicates a mode. Peak picking method is used to initial estimate of natural frequencies. Initial estimate is important to create enhanced frequency response function (EFRF) for each mode.

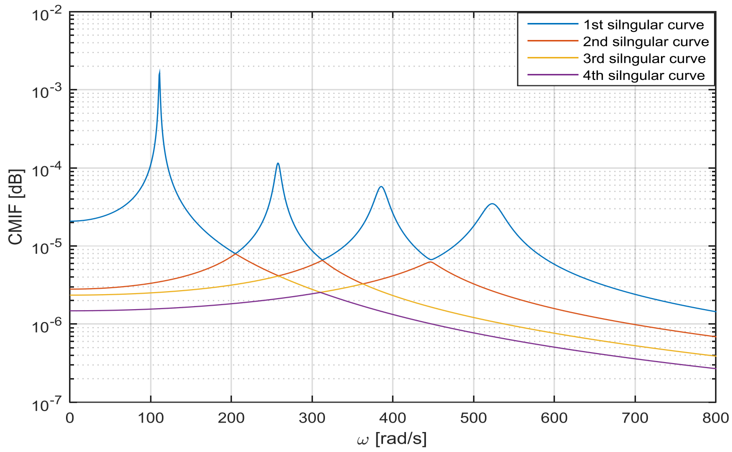

Fig. 3 CMIF plot.

The EFRF method is used to identify natural frequencies, and scaling of an equivalent single DOF characteristic that is associated with each peak in the CMIF [7, 8]. The EFRF is based on the concept of physical to modal coordinate transformation and can be defined as a weighted average of all of the measured FRFs, where the left and right singular vectors used as discrete modal filters. This is the way how the modes are isolated. Enhanced frequency response function is defined as follow: [4, 9]

$$eFRF(\omega)_r = \{U(\omega_r)\}^H [H(\omega)]\{V(\omega_r)\}, \tag{4}$$

where $eFRF(\omega)_r$ is an enhanced frequency response function of r-th mode. The EFRFs obtained for each mode of the analyzed system are shown in Fig 4.

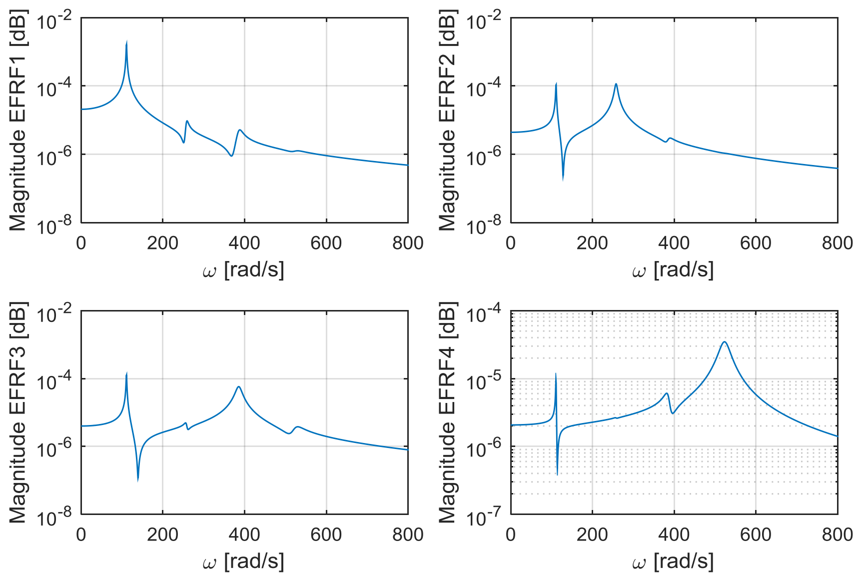

Fig. 4 EFRFs of the system.

As the next step, the rational fraction polynomial was applied to these functions to estimate modal parameters. The rational fraction polynomial is an iterative method to estimation of modal parameters from function, which describes dynamic behavior of mechanical system. This method says that frequency response function can be written as a ratio of two polynomials as follow [5, 10]

$$H(\omega) = \frac{\sum_{k=1}^{m} a_k \cdot (\omega)^k}{\sum_{k=1}^{n} b_k \cdot (\omega)^k}, \tag{5}$$

where a_k, b_k are sought unknown polynomial coefficients. These coefficients are not the modal properties direct. Unknown coefficients are achieved by minimizing the error function e_i which is defined by equation [9, 10]

$$e_i = \sum_{k=0}^{m} a_k (\omega_i)^k - h_i \left[\sum_{k=0}^{n} b_k (\omega_i)^k + (\omega_i)^n \right], \tag{6}$$

where h_i represents FRF data at frequency ω_i. Minimizing of the error function is performed by using least-squares technique. Eq. (6) can be also written for EFRF. This estimation method was applied to each of EFRFs. Modal parameters obtain by this procedure are listed in Table 2. The comparison of the results in Table 1 and Table 2 shows the good agreement between the analytical solution and the estimation process.

Table 2 Modal parameters estimated from EFRFs.

Mode	**1.**	**2.**	**3.**	**4.**
Frequency [Hz]	17.680	39.372	59.763	81.771
Damping ratio [%]	0.61	1.14	1.90	2.81

Computational efficiency of the proposed approach

The standard estimation procedure is based on the extraction of modal parameters directly from FRF matrix. This is effective only if FRF matrix is not too large, otherwise there may be problems with the time-consuming computations and lack of memory. In order to avoid these problems, the modal parameters can be estimated from Enhanced Frequency Response Functions derived from FRF matrix. This allows to include all measurement data while retaining the maximum information about the dynamic behavior of the structure. For the purpose of comparing the computational efficiency of both approaches, a special program was created in Matlab. The program generated FRF matrices with a different number of measurement DOFs and measured computational time of the estimation process. The length of discrete frequency spectrum was the same in all cases (500 spectral lines). In this study, RFP method was used as the estimation algorithm. The results of the comparison are shown in Table 3. The results show the significant differences in the duration of the computation and in the memory usage, particularly for the large FRF matrix. The modal parameter estimation from EFRFs is much more effective. The parameters are determined for a relatively short time with minimum memory usage even in FRF matrix with tens of thousands of measurement DOFs. If the modal parameters are estimated directly from FRF matrix, the computation time and the memory usage increase significantly with an increasing number of measurement DOFs. As can be seen, when FRF matrix had 5929 DOFs, the computation failed after 3 h and

49 min due to lack of memory. The proposed approach was advantageously implemented in software application DICMAN 3D [9] that is used to measure modal parameters by high-speed digital image correlation method.

Table 3 Comparison of computational time and memory usage between the standard and proposed estimation procedure.

Number of DOF of the system	**Modal parameters estimated directly from FRF matrix**		**Modal parameter estimated from EFRFs**	
	Computational time [s]	**Memory used by Matlab [%]**	**Computational time [s]**	**Memory used by Matlab [%]**
256	6.525	15.2	6.951	4.5
1024	53.96	39.6	11.89	4.6
2116	165.4	94.0	16.52	4.7
4096	1906	94.0	27.65	5.0
5929	Out of memory	-	34.03	5.1
22500	-	-	112.2	6.2
40000	-	-	209.8	7.0

Conclusions

The aim of the paper was to consider the accuracy and efficiency of modal parameter estimation process when modal parameters are extracted from enhanced frequency response functions. These functions represent responses of separated modes in modal space and they are described as the response of an equivalent SDOF system. By this way, the data contained in the FRF matrix are reduced to a few functions. It is advantageous when the matrix is relatively large. The main advantage of the proposed approach lies in a significant reduction in memory requirements and computational time while keeping a high accuracy of modal parameters.

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