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Determining the Young's Modulus of a Specimen Using Experimental Modal Analysis

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Abstract. The paper deals with determining the Young's modulus of a steel specimen that is calculated on the basis of natural frequencies determined by experimental modal analysis. The paper described two different approaches to do it. The first approach is based on analytical calculation of fundamental bending frequencies of a beam with a rectangular cross section. The paper provides results for two cases: free-free beam and fixed-free beam. The second approach is based on FEM analysis combined with parameter optimization method. This approach is applied on a free rectangular plate the analytical solution of which is more complicated.

Introduction

We know a lot of engineering problems where the exact knowledge of material parameters such as Young's modulus or Poisson ratio is essential for their solution or for an accuracy of numerical models. The material parameters are commonly determined by standard material tests. Alternatively, they can be determined by experimental modal analysis where the measured natural frequencies are subsequently used for an analytical solution. In the case of isotropic materials, it is preferable to perform an analysis of bending beam vibration because it is very easy to obtain accurate analytical solution for different boundary conditions [1, 2]. In general, it is recommended to analyze a free-free beam because the real stiffness of constraints is quite difficult to take into account in the calculation. Moreover, we can very well model a free suspension. In the case of orthotropic materials, the solution of plate vibration is essential [4]. Unfortunately, there exists no closed form solution for the case of a rectangular Kirchhoff plate with free boundaries, but several approximate methods have been proposed and applied. Warburton [4] used characteristic beam vibration functions in Rayleigh's method [5] to obtain a useful and simple approximate expression for the natural frequencies of vibration of thin isotropic plates. His work was extended by Hearmon [6] and applied to special orthotropic plates and again by Dickinson [7] to include the effect of uniform in-plane loads. Kim and Dickinson [8] provided an improved approximate expression, where they use Rayleigh's method in connection with the minimum potential energy theorem. Iguchi [9] gave solutions for an isotropic rectangular plate, however, he limits his determination to squared plates only. Leissa [10] presented comprehensive and accurate analytical results for the free vibration of rectangular plates. He applied the Ritz method [11] and compared the results with the method of Warburton [4]. However, his work is limited to isotropic plates. Leissa's work was extended by Deobald and Gibson [12] who applied the Rayleigh Ritz method to orthotropic plates as well. Gorman [13, 14] solved the differential equation for isotropic as well as orthotropic plates using a superposition method, which allows one to full fill the boundary conditions with desired accuracy. Wang and Lin [15] presented a systematic analysis for solving boundary value problems in structural mechanics, where a weighted residual form of the differential equations is used with sinusoidal weighting functions. Recently, this approach has been extended for calculating the eigenfrequencies and eigenmodes of an orthotropic plate with completely free boundaries using an exact series solution by Hurlebaus et al. [16]. As the quasi-analytical methods can be used the concept of sinusoidal equivalent length, the one term Rayleigh method, the three term Rayleigh method, the Rayleigh-Ritz method, the superposition method, and the exact series solution [17].

Procedures for obtaining Young's modulus

A. Beam vibration

The equation of motion for the bending vibration of a beam with constant cross-section and material properties is given by [16]

$$c^2 \frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \qquad \rightarrow \qquad c^2 \frac{\partial^4 w(x,t)}{\partial x^4} - \omega^2 w(x,t) = 0\,, \tag{1}$$

where $w(x,t)$ is the transverse vibration of the beam at position x at time t, ω is the natural frequency of the beam, and c is given by

$$c = \sqrt{\frac{EI}{\rho A}}\,, \tag{2}$$

in which I is the cross-sectional area moment of inertia, ρ is the mass per unit volume, and A is the cross-sectional area. The natural frequencies of the beam are from

$$\omega_n = (\beta_n \ell)^2 \sqrt{\frac{EI}{\rho A \ell^4}}\,, \tag{3}$$

where the subscript n is the bending mode number and β_n is the constant depending on boundary conditions (see Tab. 1). Young's modulus E of the beam material can be calculated from Eq. (3) as follows

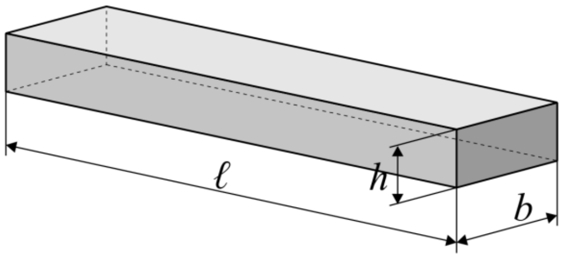
$$E = \frac{\omega_n{}^2 \rho A \ell^4}{(\beta_n \ell)^4 I}\,. \tag{4}$$

Tab. 1 Boundary condition values for the first four bending modes of the beam

Support condition	$(\beta_1 \ell)$	$(\beta_2 \ell)$	$(\beta_3 \ell)$	$(\beta_4 \ell)$
Free - Free	4.7300	7.8532	10.9956	14.1371
Clamped - Free	1.8751	4.6941	7.8539	10.9956

The natural frequencies of the beam were determined by experimental modal analysis. The object of measurement was the beam with rectangular cross section made of steel. The dimensions of the beam and calculated geometric characteristics and material properties are listed in Tab. 2. The mass of the beam was determined by weighing.

Tab. 2 Dimensions and calculated geometric characteristics and material properties of the beam

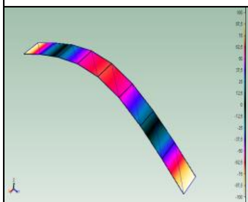
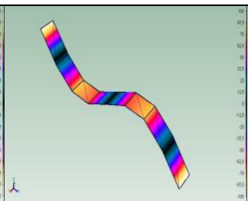
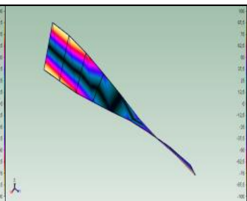
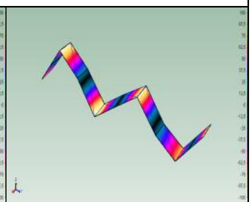
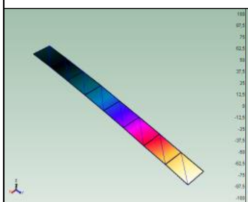
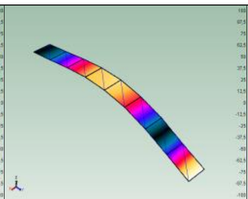
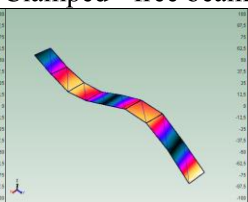
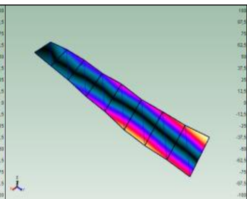
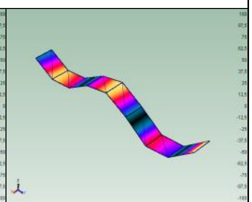
Dimensions	Geometric characteristics
$\ell = 0.401$ m $b = 0.040$ m $h = 0.008$ m  	$A = bh = 3.2\cdot 10^{-4}\ \mathrm{m}^2$ $I = \dfrac{bh^3}{12} = 1.7067\cdot 10^{-9}\ \mathrm{m}^4$
	Material properties
	$m = 1.004\,\mathrm{kg}$ $\rho = \dfrac{m}{A\ell} = 7832\,\mathrm{kg}\cdot\mathrm{m}^{-3}$

Two types of support conditions of the beam were investigated: free-free and clamped-free (Fig. 1). The free-free supporting was realized by two elastic ropes. The experimental modal analysis was performed using the measurement system Pulse LAN XI. The responses were measured at one point by laser Doppler vibrometer Polytec PDV-100. The beam was excited at 18 points by impact hammer Bruel&Kjaer 8206 with aluminum tip.

Fig. 1 Measurement setup for free-free and clamped-free supporting of the beam

Rational Fraction Polynomial-Z method was used to estimate modal parameters. In both cases, the first 5 modes were found, including 4 bending modes and 1 torsion mode. Their natural frequencies and mode shapes are shown in Tab. 3.

Tab. 3 Natural frequencies and mode shapes of the beam

Free - free beam				
				
1: 265,44 Hz	2: 731,27 Hz	3: 1429,97 Hz	4: 1496,95 Hz	5: 2355,81 Hz
Clamped - free beam				
				
1: 40,528 Hz	2: 252,50 Hz	3: 699,08 Hz	4: 716,36 Hz	5: 1333,02 Hz

After substituting the values from Tab. 2 and the natural frequencies of bending modes into Eq. 4, Young's modulus of the beam material was calculated. Its values are listed in Tab. 4.

Tab. 4 Young's modulus of the beam

Support condition	1. mode	2. mode	3. mode	4. mode	Mean value
Free - Free	211.81 GPa	210.76 GPa	209.69 GPa	208.28 GPa	210.14 GPa
Clamped - Free	199.19 GPa	196.84 GPa	192.54 GPa	182.21 GPa	192.70 GPa

Since the analytical solution is expressed for the ideal boundary conditions and does not take into account the real stiffness of the support, Young's modulus values determined for the free supported beam are more in line with the expected value than the values determined for the clamped beam. From this point of view, it is more preferable to determine material constants on the basis of natural frequencies of a free supported specimen. The conditions of this type of support can be achieved relatively easily by hanging the specimen with elastic ropes or by laying it on a foam pad. A necessary condition is that the frequency of the last rigid body mode must be at least ten times less than the frequency of the first flexible mode [18].

B. Isotropic plate vibration

The second procedure is based on finite element analysis that is combined with parametric optimization. The object of interest was a rectangular plate of constant thickness made of steel. The plate was considered as free on all edges. The free support was realized by putting the plate horizontally on three stretched elastic ropes. The responses were measured at one point by laser Doppler vibrometer Polytec PDV-100. The plate was excited at 54 points by impact hammer Bruel&Kjaer 8206 with aluminum tip. The measurement setup and the basic dimensions of the plate can be seen in Fig. 2.

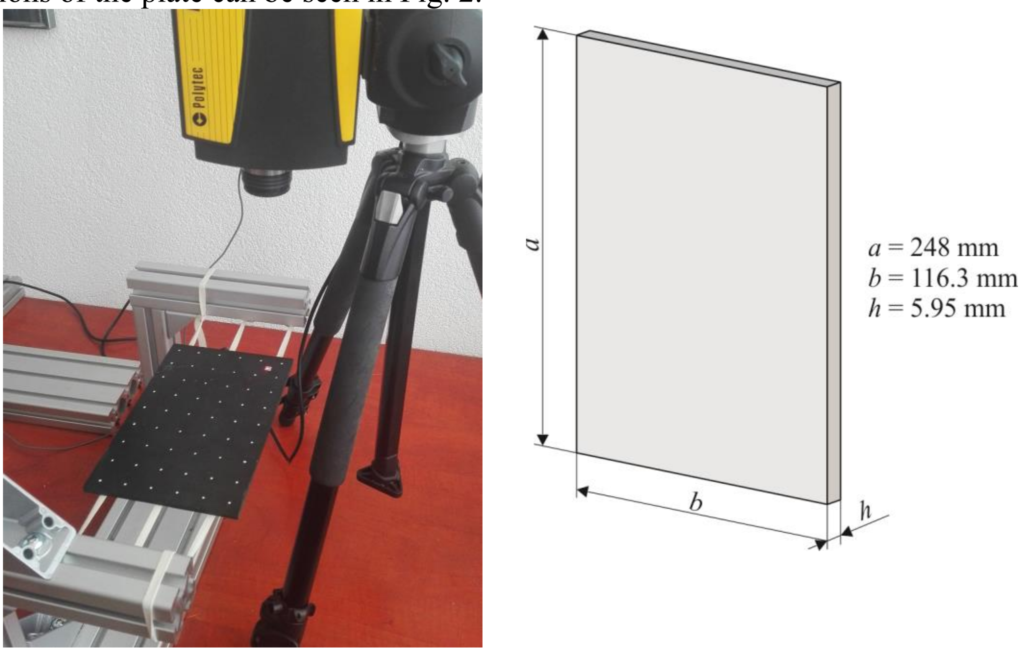


Fig. 2 Measurement setup and the basic dimensions of the plate

Modal parameters of the plate were determined by Rational Fraction Polynomial-Z method. Fig. 3 shows the plot of Complex Mode Indicator Function peaks of which correspond to the natural frequencies. The first peak represents rigid body mode. The natural frequencies and mode shapes of flexible modes are shown in Tab. 5.

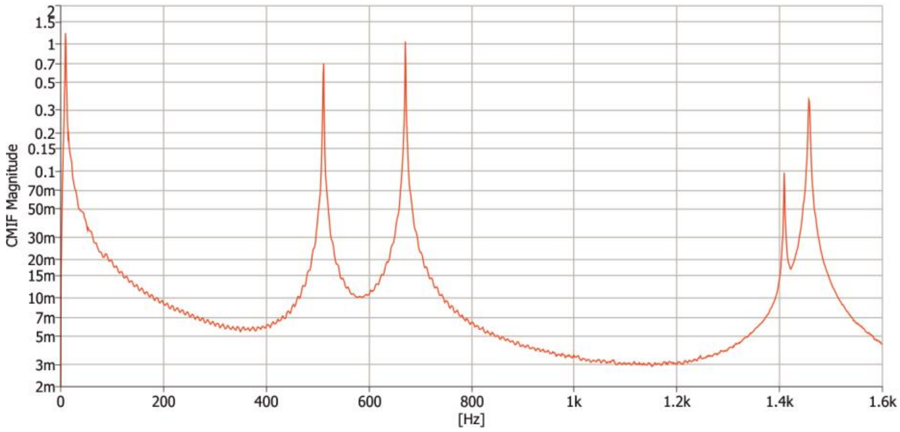
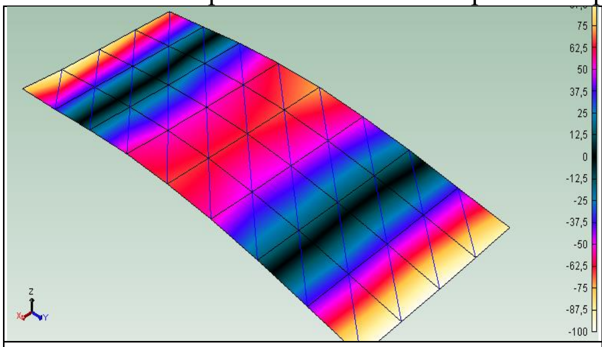
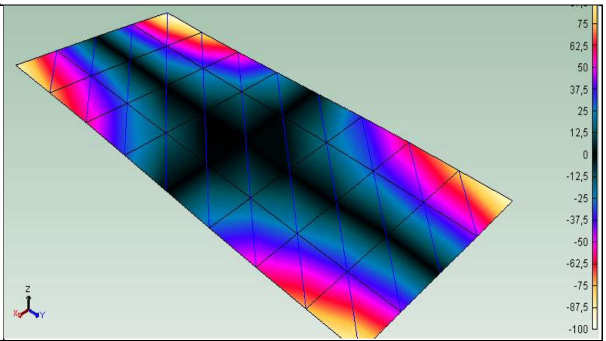
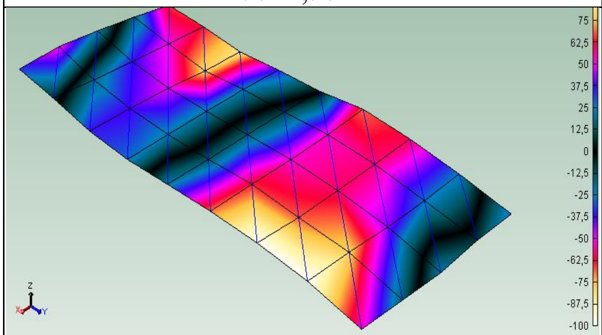
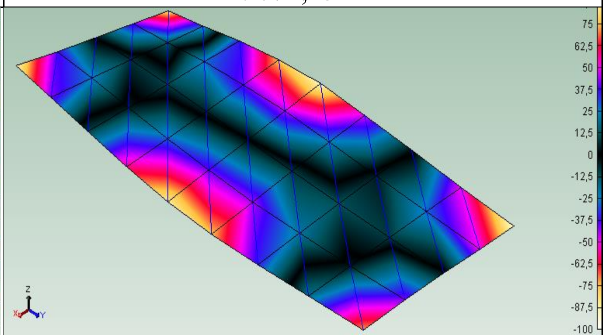


Fig. 3 CMIF plot of the plate

Tab. 5 Natural frequencies and mode shapes of the plate

	
1: 511,58 Hz	2: 671,15 Hz
	
3: 1409,09 Hz	4: 1457,48 Hz

As mentioned above, Young's modulus value was estimated numerically by parametric optimization. The numerical solution was carried out in NX Nastran software. The optimization process was based on eigenvalue solution provided by SOL103 Real Eigenvalues.

As the first, 2D CAD model of the plate was created. Shell elements CQUAD8 of approximated width 4 mm were used to mesh the model. Subsequently, the material properties were associated to FE model. The material density of the plate was determined by calculation as follows

$$\rho = \frac{m}{abh} = \frac{1.324}{1.7161 \cdot 10^{-4}} = 7715 \ \ \mathrm{kg} \cdot \mathrm{m}^{-3},$$

where m is the mass determined by weighing. The initial Young's modulus value was set to 150 GPa. Poisson's ratio was 0.29. There were defined no boundary conditions.

Geometry optimization module was used in the optimization process. Young's modulus was defined as a design variable that can acquire values from 100 GPa to 300 GPa. The objective function was the natural frequency of chosen mode, so the optimization process was carried out 4 times. Fig. 4 shows the course of the optimization where target function was the frequency of the first mode 511.58 Hz.

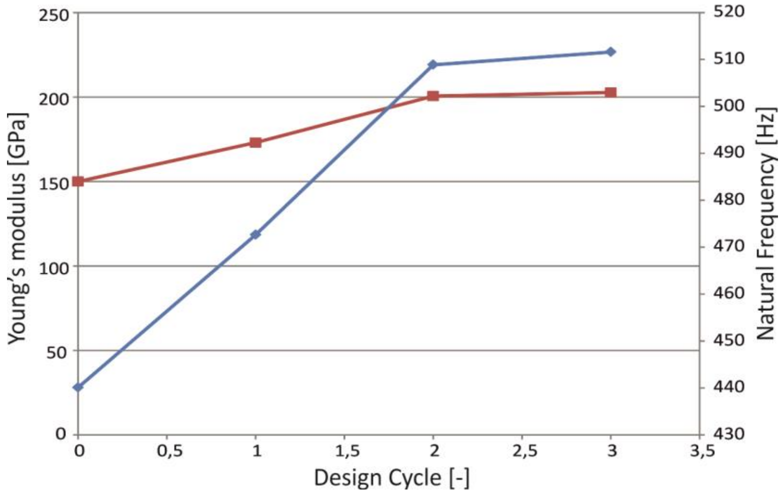


Fig. 4 Target and design variable depending on design cycle

The results of the optimization process for each target function are listed in Tab. 6. The values of Young's modulus are in the range from 200.81 to 205.63 GPa. Its mean value is 203.45 GPa. Tab. 7 shows the absolute percentage error of the natural frequency values. The highest mean absolute percentage deviation (MAPD) 0.65% was reached in the third optimization process. However, such deviations are entirely acceptable. The first four mode shapes obtained by FEM analysis are shown in Fig. 5.

Tab. 6 Results of the optimization process

Target value of objective function	1. mode	2. mode	3. mode	4. mode	E [GPa]
1. mode: 511,58 Hz	**511.59 Hz**	666.37 Hz	1415.73 Hz	1450.53 Hz	202.70
2. mode: 671,15 Hz	515.28 Hz	**671.16 Hz**	1425.90 Hz	1460.96 Hz	205.63
3. mode: 1409,09 Hz	509.20 Hz	663.25 Hz	**1409.11 Hz**	1443.75 Hz	200.81
4. mode: 1457,48 Hz	514.04 Hz	669.55 Hz	1422.49 Hz	**1457.45 Hz**	204.64
				Mean value:	203.45

Tab. 7 Percentage error and mean absolute percentage deviation of the natural frequency values

Target value of objective function	1. mode	2. mode	3. mode	4. mode	MAPD
1. mode: 511,58 Hz	0.00 %	0.71 %	0.47 %	0.48 %	0.41 %
2. mode: 671,15 Hz	0.72 %	0.00 %	1.19 %	0.24 %	0.54 %
3. mode: 1409,09 Hz	0.46 %	1.19 %	0.00 %	0.95 %	0.65 %
4. mode: 1457,48 Hz	0.48 %	0.24 %	0.95 %	0.00 %	0.41 %

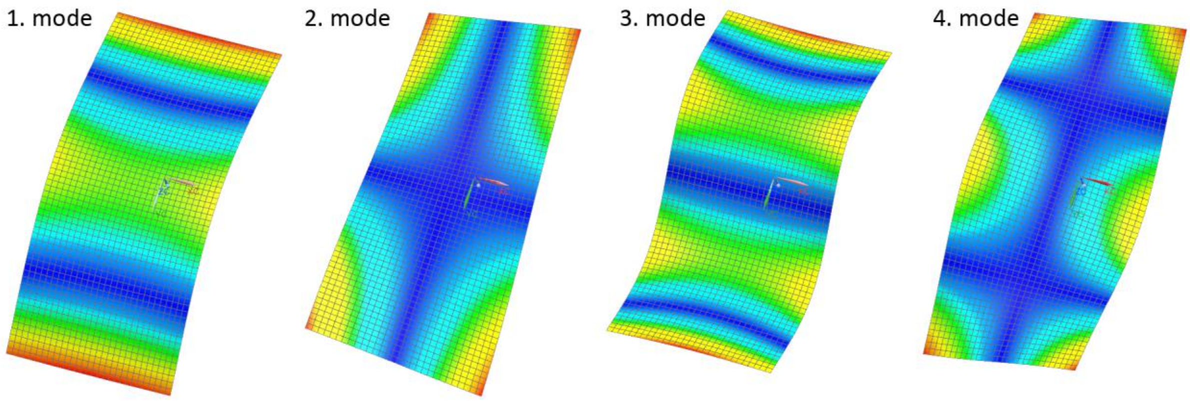


Fig. 5 The first four mode shapes of the plate obtained by FEM analysis

Conclusions

The paper presented two methods to determine Young's modulus of an isotropic material. The first of them is a combination of experimental modal analysis and analytical solution, in which the known values of natural frequencies are substituted into the well-known equation for calculation of bending mode frequencies of a beam. It has been shown that the free support of the beam is more advantageous for determining Young's modulus value. The second method uses the parametric optimization that is applied on FE model of a plate and the eigenfrequency of a given mode is used as an objective function to calculate Young's modulus. The obtained results confirm that this approach is sufficiently precise and effective.

Acknowledgement

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