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Materials Selection for Efficient Cross-section of Components

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Abstract. Materials selection is an essential part of the product design process as materials influence most of the product's properties. Currently design engineers are able to choose from up to 160 000 engineering materials. We use the methodology of prof. Ashby and CES EduPack software to material selection.

Introduction

This paper introduces methods for co-selecting material and section shape. In many applications section shape is not a variable. But when components carry bending, torsion or compressive loads, both the area and shape of the cross-section are important. By shape we mean that the cross-section is formed to a tube, I-section or the like. Efficient shapes use the least material to achieve a given stiffness or strength. So material and shape are coupled, requiring a method of choosing them together.

However, shape factors are only developed for simple loading states. Derivation of shape factors for combined loads is one of the aims of this paper.

Selection of Material and Shape

Shape Factor. We have a many types of profiles as I-sections, rods, tubes, etc. which we can use for designing, but which one is the best? That depends on three things, type of load, shape of the profile and material. For this purpose we will neglect the material because we will compare only the profiles with the same material. Loads we can divided into 5 categories, tension, compression, torsion, bending and shear but we will work only with compression, torsion and bending because in tension and shear shape of the profile is not important. Also as in the previous case, we will compare only the profiles with the same loads.

Every profile have a volume defined by a cross section and length. Together with the density of the material we can get a weight of the profile. If we want compare two different shapes of the profiles, we need to do for the constant weight. As we mentioned above, the material is constant and also the cross section and length must be constant too.

From previous information we can see that the only variable which we will use, will be the shape of the cross section of the profiles which is define by dimensions. By comparison of two different shapes we can get the shape factor which is dimensionless number and this number expresses how much is the compared shape better (bigger than 1) or worse (lower than 1) than the reference one.

For every comparison is important to choose the reference shape with which we will compare the new one. As the best option is a solid square rod with cross section area A_0.

If we want to make a comparison of two different cross sections, then firstly we need to say if we are searching shape factor for elastic loading or shape factor for onset of plasticity (failure). For elastic loading we need to compare second moment of area, for failure we need to compare section modulus of area.

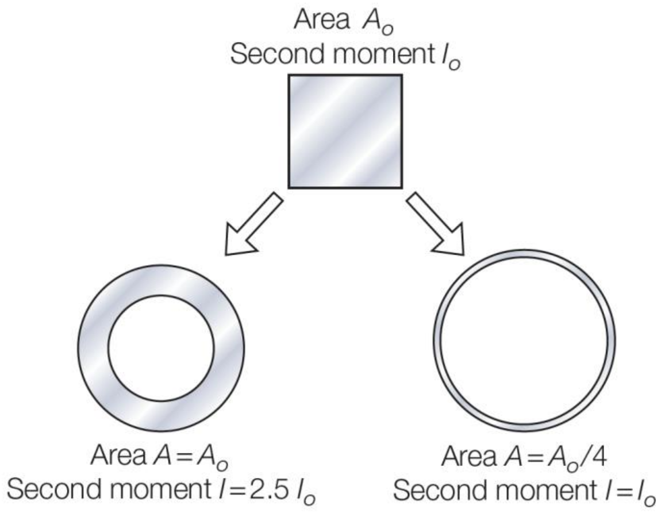


Fig. 32 Principle of the shape factor [1]

As we can see in the Fig. 1, when we compare the shape of the solid square rod with the shape of the tubes we can get different results. The tube in the left has the same cross section area as the square rod but it is 2.5 times stiffer. The tube in the right has the same stiffness as the square rod but it is 4 times lighter.

As we mentioned in previous chapter, we have two types of shape factors, the shape factor for elastic loading and shape factor of failure.

The shape factor for elastic bending (ϕ_B^e) is define as a ratio of bending stiffness of solid square rod (S_0) and compared profile (S) where:

$$S \propto \frac{E \cdot I}{L^3} \tag{1}$$

Here E is Young's modulus, I is the second moment of area and L is length.

$$\phi_B^e = \frac{S}{S_0} = \frac{\frac{E \cdot I}{L^3}}{\frac{E \cdot I_0}{L^3}} = \frac{I}{I_0} \tag{2}$$

$$I_0 = \frac{{b_0}^4}{12} = \frac{A^2}{12} \tag{3}$$

$$\phi_B^e = \frac{12 \cdot I}{A^2} \tag{4}$$

The factor is dimensionless number that mean that the value of the factor does not depend on scale, only on shape.

The shape factor of failure for bending (ϕ_B^f) we can get from equation for bending stress (σ_b). Bending stress is a ratio of the bending moment (M_B) and the section modulus (Z). If we compare the bending cross section modulus for solid square rod (Z_0) with the bending cross section modulus of any other shape (Z) we will gain the shape indicator for strength efficiency of the shaped beam ϕ_B^f.

$$\sigma_B = \frac{M_B}{Z} \leq \sigma_{B_{max}} \tag{5}$$

$$z_0 = \frac{b_0 \cdot h_0}{6} \tag{6}$$

Because Z_0 is section modulus for square rod, then $b_0 = h_0$ and then:

$$z_0 = \frac{{b_0}^3}{6} = \frac{A^{\frac{3}{2}}}{6} \tag{7}$$

$$\phi_B^f = \frac{Z}{Z_0} = \frac{6 \cdot Z}{A^{\frac{3}{2}}} \tag{8}$$

Similar shape factors characterize stiffness and strength in torsion. For elastic twisting:

$$\phi_T^e = 7.14 \cdot \frac{K}{A^2} \tag{9}$$

And for failure in torsion:

$$\phi_T^f = 4{,}8 \cdot \frac{Q}{A^{\frac{3}{2}}} \tag{10}$$

Where K is torsional moment of area and Q is torsion cross section module.

Maximum values for shape factors depends on materials and we can gain theoretical maximum values from equation:

$$\phi_e = 2\sqrt{\frac{E}{\sigma_y}} \tag{11}$$

Where E is Young modulus and σ_y is Yield strength. Maximum theoretical values of shape factors for few basic material are listed in the Table 1.

Tab. 3 Limits for Shape factors [1]

Material	**Max ϕ_e**	**Max ϕ_f**
Steels	65	13
Aluminium alloys	44	10
GFRP and CFRP	39	9
Unreinforced polymers	12	5
Woods	8	3
Elastomers	< 6	-

Material Indices that Include Shape

This paper introduces methods for co-selecting material and section shape. We use the methodology of prof. Ashby and CES EduPack software to material selection [2]. The methodology of materials selection is based on material indices. This chapter explains the ground of the material indices and their interconnection with the shape factors. A wider explanation is in the book [1].

Materials selection is based on and driven by engineering design process of a given product. The requirements on the product are the inputs for this process. The requirements can be divided into three categories: function, constraints, goals and free variables. The function is defined by the purpose of the part, e.g. to support load. The constraints are conditions that must be met, e.g. maximum deflection or maximum dimensions. During the design process there are some goals to be achieved. We might want the product to be as light or as cheap as possible. Some parameters might be adjusted to maximize the fulfilment of goals. These are free variables.

In Fig. 2 engineering design-driven materials selection scheme is shown. Design requirements are translated into product specification suitable for materials selection. This can be done by deriving of material indices. The constraints set out limit values of certain properties. The goals define the material indices for which we seek extreme values. If a goal is not bound with a constraint the material index becomes a simple material property. Otherwise the index becomes a group of properties.

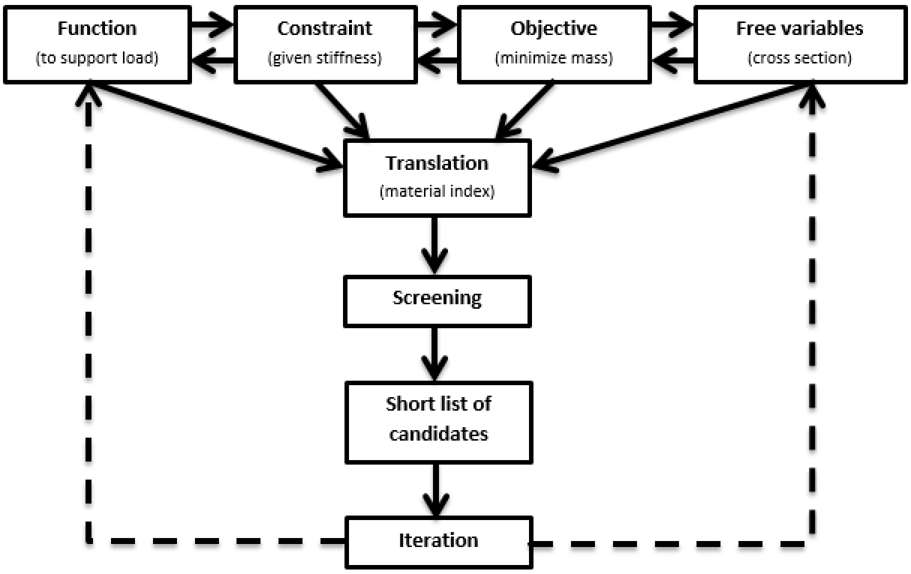


Fig. 33 Engineering design-driven materials selection

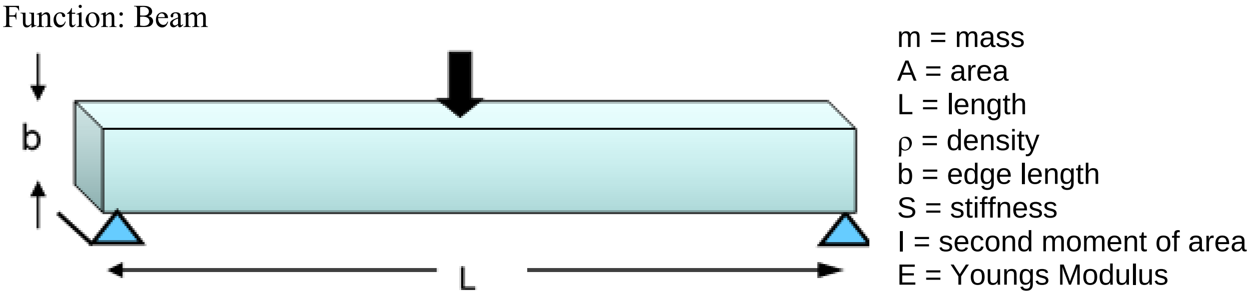
The performance of a part is given by three parameters: functional requirements (F), geometrical dimensions (G) and material properties (M). It can by expressed by the equation below:

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M) \tag{12}$$

The performance for is maximized when we maximize $f_3(M)$ which is called material index. Each combination of function, goal and constrain leads to a material index which makes this method general an applicable for a wide range of problems.

Material index for a stiff, light (bending) beam. The most usual mode of loading of engineering structures is bending. All these structures carry bending moments, they are beams. The requirement here is for a beam of specified stiffness and minimum mass.

Function: Beam



Constraints:
Length L is specified (geometric constraint)
Bending stiffness is specified as S* (functional constraint)

$$S = \frac{CEI}{L^3} = \frac{CEA^2}{12L^3} \geq S^* \tag{13}$$

$$I = \frac{b^4}{12} = \frac{A^2}{12} \tag{14}$$

Objective:
Minimize mass m of the beam

$$m = A\,L\,\rho \tag{15}$$

Performance metric m:

$$m = \left(\frac{12L^5S}{C}\right)^{\frac{1}{2}} \left(\frac{\rho}{E^{\frac{1}{2}}}\right) \tag{16}$$

Material index for light, stiff beam is:

$$\left(\frac{E^{\frac{1}{2}}}{\rho}\right) \tag{17}$$

This is material index that does not depend on shape.

Material index that include shape. But stiffness and strength-limited design depends on shape of cross-section of components. For stiff, light beam material and shape are coupled, requiring a method of choosing them together.

For the square beam of

$$I_0 = \frac{A^2}{12} \tag{18}$$

whereas for the shape beam

$$I = \frac{\phi_e \cdot A^2}{12} \tag{19}$$

The resulting equation for the metric m shows how the mass of the beam depends on the mechanical constraint it must meet (S) on the section shape (ϕ_e) and on the material of which its made (ρ/E1/2). The quantity (ρ /$(\phi_e E)^{1/2}$) can be thought as a "shaped-material index".

Material index that include shape for light, stiff (bending) beam is:

$$\left(\frac{\phi_e E^{\frac{1}{2}}}{\rho}\right) \tag{20}$$

Selecting material-shape combinations

For mechanical designers is the most often goal of designing to reach a minimum mass of construction. From this reason is useful to connect shape factor indicator with material indicator together. If we do this, we will get the indices as you can see in the Tab. 2.

Tab. 4 Indices with shape [1]

Component Shape, Loading, and Constraints	Stiffness-limited Design*	Strength-limited Design*
Tie (tensile member) Load, stiffness, and length specified, section-area free	$\frac{E}{\rho}$	$\frac{\sigma_f}{\rho}$
Beam (loaded in bending) Loaded externally or by self-weight, stiffness, strength and length specified, section area and shape free	$\frac{(\phi_B^e E)^{1/2}}{\rho}$	$\frac{(\phi_B^f \sigma_f)^{2/3}}{\rho}$
Torsion bar or tube Loaded externally, stiffness, strength, and length specified, section area and shape free	$\frac{(\phi_T^e E)^{1/2}}{\rho}$	$\frac{(\phi_T^f \sigma_f)^{2/3}}{\rho}$
Column (compression strut) Collapse load by buckling or plastic crushing, strength and length specified, section area and shape free	$\frac{(\phi_B^e E)^{1/2}}{\rho}$	$\frac{\sigma_f}{\rho}$

For example, materials for stiff, shaped beams of minimum weight:

- Fixed shape (φe fixed): choose materials with low $\frac{\rho}{E^{\frac{1}{2}}}$
- Shape φe a variable: choose materials with low $\frac{\rho}{(\phi_e \cdot E)^{\frac{1}{2}}}$

Tab. 5 Suitability of materials [1]

Material	$\rho\ [Mg/m^3]$	$E\ [GPa]$	$\phi_{e,max}$	$\rho/E^{\frac{1}{2}}$	$\rho/(\phi_{e,max} \cdot E)^{\frac{1}{2}}$
1020 Steel	7,85	205	65	0,55	0,068
6061 T4 Al	2,70	70	44	0,32	0,049
GFPR	1,75	28	39	0,35	0,053
Wood (oak)	0,9	13	8	0,25	0,088

For fixed shape (up to $\phi_e = 8$) is wood the best solution. For maximum shape ($\phi_e = \phi_{e,max}$) are Al-alloys the best solution. Steel recovers some performance through high $\phi_{e,max}$.

Shape factors created by numerical analysis

Theory of material indices developed by professor Ashby assumes developing of indices based on analytical solution [1]. It needs to know analytical (mathematical equations) model of a chosen physical phenomenon. But in many cases, only numerical solution is known or it is difficult to create analytic solution or arrange parameters during process of creation a material index or shape factor.

The method of creating shape factors using numerical solution is based on analysis of FEM solver output data. During FEM output analysis, changes of a solved object parameters against changes of calculated deformation or stress are compared. The aim of comparison is to find equation of a factor using regression, logarithm or elementary equations. This paper describes creation of shape factors of fixed beam with rectangle cross section.

First step of analysis is to define scope of analysis:

- Function: beam, one end fixed, other end free and loaded by moment (bending, torsion or its combination)
- Objective: minimize stress (von Mises)
- Constrains: linear elastic material, length
- Free variables: width, height, load

Tab. 6 Analysis parameters

Parameter	Symbol	Type	Bending factor	Torsion factor	Combined factor
Width	B	Shape parameter	Variable	Variable	Variable
Height	H	Shape parameter	Variable	Variable	Variable
Length	L	Constant	Constant	Constant	Constant
Material properties	E, υ	Constant	Constant	Constant	Constant
Bending moment	M_b	Load	Variable	0	Variable
Torsion moment	M_t	Load	0	Variable	Variable

First step is modelling a parametric geometry of the beam. In this case, Design Modeller application from ANSYS bundle was used. The geometry was linked to Static Structural module. Material of beam must be linear elastic, steel was selected. Application of loads and constrain is described on the picture (Fig. 3):

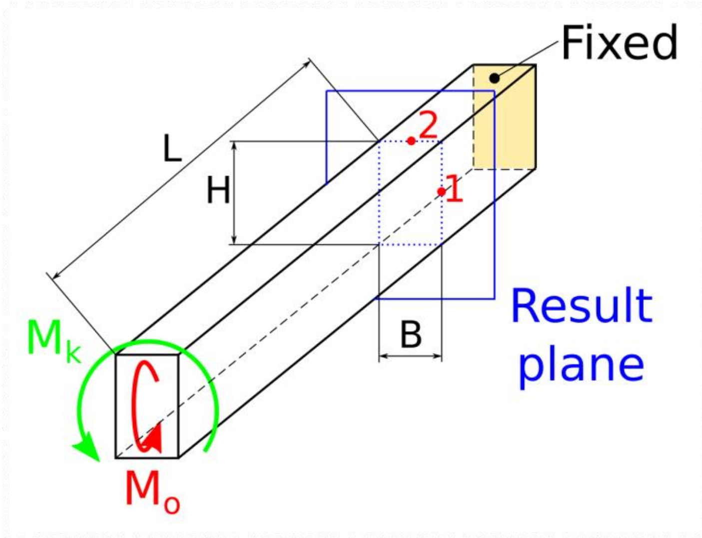


Fig. 34 Loads and constrains in FEM analysis

The analysis does not consider the effect of fixed constrain. Parametric model of the beam is a little bit longer than considered, but stress results were calculated at a result plane. The result plane is normal to beam axis and its distance from free end of the beam is the same as considered length L of a beam (Fig. 3).

It is also necessary to create design points, which are used to changes of simulation parameters. Design points must be created according change of the shape of cross section and table Tab. 4 above. Shape of rectangle cross section is defined by equation:

$$shape = \frac{H}{B} \tag{21}$$

For all cases in this paper:

$$H \geq B \tag{22}$$

It is necessary to eliminate influence of cross section size, so the cross section dimensions should pass the condition:

$$H_1 B_1 = H_2 B_2 = A_0 \tag{23}$$

Simulations of examples bellow were calculated using FEM system ANSYS Workbench 17. Result analysis of FEM output was done in Python 3 with math libraries developed under SciPy project.

Shape factor for pure bending. Creation of bending shape factor is relatively simple, because there is direct dependence of normal stress on the dimensions of cross section.

ANSYS output data was analyzed using equation:

$$\lambda_p = \ln \frac{\sigma_{i+1}}{\sigma_i} \cdot \left(\frac{p_{i+1}}{p_i}\right)^{-1} \tag{24}$$

where p is parameter which influence is analyzed (in this case B or H) and λ_p its estimated exponent. The result of analysis of FEM output are values:

$$\lambda_B = 1 \tag{25}$$

$$\lambda_H = -1 \tag{26}$$

Estimated shape factor should be something as:

$$\psi = B^{\lambda_B} H^{\lambda_H} = \frac{B}{H} \tag{27}$$

where ψ is estimated solution for bending factor. It is necessary to verify slope of equation of estimated factor ψ:

$$\psi = B^{\lambda_B} H^{\lambda_H} = \frac{B}{H} \tag{28}$$

where λ_s is slope correction exponent and its value is:

$$\lambda_s = 0.5 \tag{29}$$

Result of previous analysis is *shape* factor *for beam loaded by bending moment* (in physics mean, lower value is better design):

$$\zeta_1 = \psi^{\lambda_s} = \left(\frac{B}{H}\right)^{0.5} = \left(\frac{H}{B}\right)^{-0.5} \tag{30}$$

The shape factor of bended beam was created from numeric values, in this case better design has lower value of shape factor (lower stress). According to prof. Ashby's theory [1], it was transformed to get the highest value for the best design:

$$\phi_B^f = \zeta_1{}^{-1} \tag{31}$$

$$\phi_B^f = \sqrt{\frac{B}{H}} \tag{32}$$

where ζ sign is shape factor with physics mean (better design has lower factor values) and ϕ sign the prof. Ashby form of shape factor (better design has highest factor value). Prof. Ashby published the same form of shape factor for beam loaded by bending moment [1].

Shape factor for pure torsion. Torsion shape factor was created similarly as bending shape factor. After preliminary analysis, solution was estimated in form:

$$\chi = \frac{H}{B} + 1 \tag{33}$$

The result of second analysis is *shape factor for the beam loaded by torsion* (in physics mean, lower factor value is better design):

$$\zeta_2 = \left(\frac{H}{B} + 1\right)^{\frac{3}{7}} \tag{34}$$

Torsion shape factor modified to prof. Ashby form:

$$\phi_T^f = \zeta_2{}^{-1} = \left(\frac{H}{B} + 1\right)^{-\frac{3}{7}} \tag{35}$$

Prof. Ashby published this factor with different equation [1]:

$$\phi_{T\,Ashby}^f = 1.6 \cdot \sqrt{\frac{B}{H}} \cdot \frac{1}{1 + 0.6 \cdot \frac{B}{H}} \tag{36}$$

Comparison of values of prof. Ashby's version of shape factor and torsion shape factor based on FEM (booth in physical mean, Fig.4):

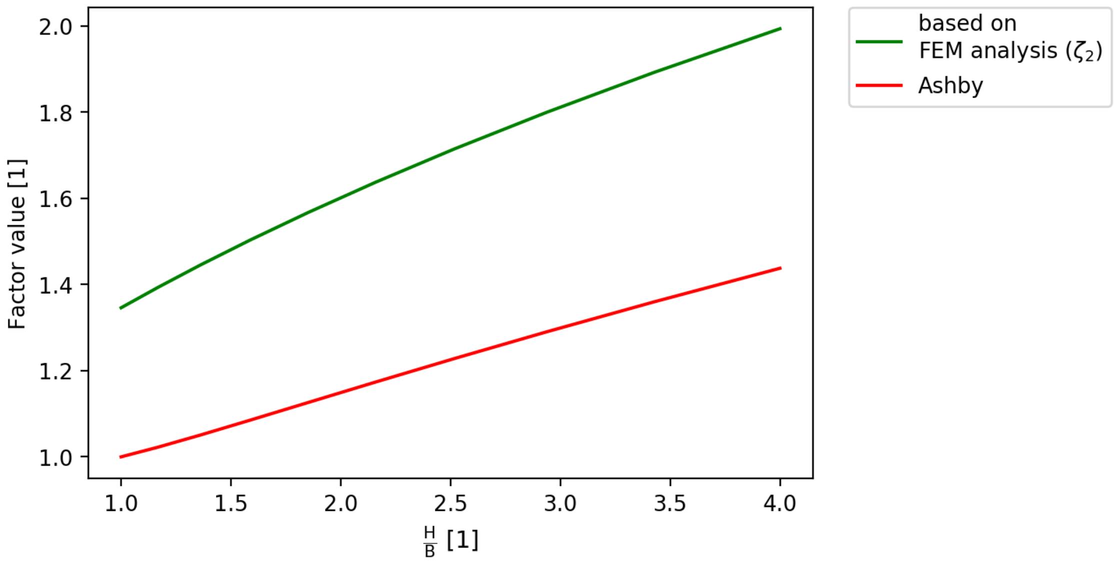


Fig. 35 Comparison of values of prof. Ashby and FEM factors

One of the properties of shape factors is presented below - ability to determine stress of a loaded object. Suppose that there is any beam loaded by torsion moment, maximal shear stress τ_1 is known. The goal is to create a new better design. Shear stress τ_2 of a new design should be determined using equation:

$$\tau_2 = \tau_1 \cdot \frac{\phi_{T2}^f}{\phi_{T1}^f} \tag{37}$$

Tab. 7 Stress prediction comparison of prof. Ashby and FEM factors

Design	B	H	factor type	factor value	Calculated stress (FEM)	Stress estimated by factor
Old	20 mm	80 mm	Ashby	1.437	192 kPa	-
			FEM	1.993		-
New	34.3 mm	46.6 mm	Ashby	1.050	139 kPa	140.3 kPa
			FEM	1.445		139.2 kPa

In this example (Tab. 5), booth shape factors has similar results. Unlike the previous example, there are two different equations, each of them provides different values. It is a question if this phenomenon is desirable due to the current state of the theory, where shape factors are compared against a given base geometry during creation [1]. Analysis of FEM data is based on another principle – comparison changes of stress result against given input parameters. However, the next example shows benefit of some relationship of FEM based shape factor value on value of calculated stress.

Combined bending-torsion shape factor. Creation of combined bending-torsion shape factor is more difficult than previous shape factors. The reason is stress distribution of rectangle cross section shape of the beam. There are two different types of stress in two areas. Let's see on the picture bellow (Fig. 5). There are two areas market by point 1 and 2. Highest stress could be in one of them or booth, depending on values of bending moment and torsion moment.

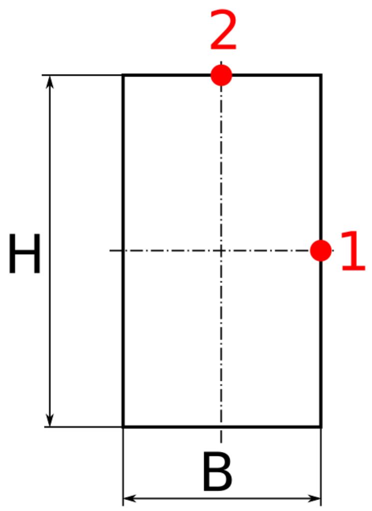


Fig. 36 Areas of higher stress

In the case of dominant bending moment (or pure bend), it is important to evaluate normal stress and shear stress in the point 2. For dominant torsion moment (or pure torsion) of the beam, there is shear stress in the point 1. In this case, von Mises stress in point 1 is higher than stress (von Mises) in point 2. The situation is explained on the picture (Fig. 6) bellow.

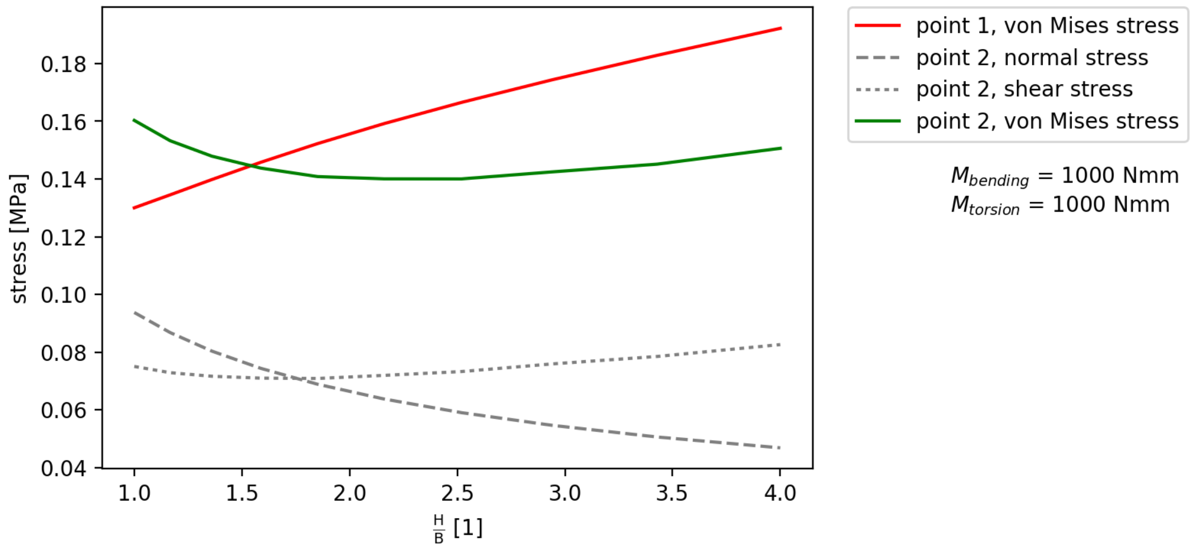


Fig. 37 Stress in point 1 and 2

The solution of this problem is a little bit complicated, but can be solved by von Mises yield criterion. Shear stress in point 1 is described by shape factor ζ_2. Shape factor ζ_1 describes normal stress in point 2. It remains to create shape factor for shear stress in point 2. From the picture (Fig. 6) above is evident, that shear stress in point 2 drops (when shape ratio of cross section grows). There is minimum of shear stress around 1.6 value of cross section shape ratio. Then shear stress increases (if shape ratio still grows). It seems an equation that describes this phenomenon could be:

$$\omega = \left(\frac{H}{B} - C\right)^2 \tag{38}$$

As mentioned, von Mises yield criterion will be used. To be used correctly, all (particular) shape factors (ζ_1, ζ_2, ζ_3) must be normalized. It means values of all shape factors must have the same relative ratio against stresses described by them. Factors ζ_1 and ζ_2 have relative ratio 10 (compared to von Mises stress), it means they are 10 times higher than stresses described them, if the beam is loaded by 1000 N.mm bending or torsion moment. Particular factors ζ_3 should be created with the same relative ratio requirement. After some analysis, final version of shape factors ζ_3 was created:

$$\zeta_3 = \frac{\frac{H}{B} + \left(\frac{H}{B} - 2\right)^2}{5 \cdot \left(\frac{H}{B} + 1\right)} + 1.1 \tag{39}$$

Normalization is explained in the picture (Fig. 7) below, all stresses were multiplied by 10:

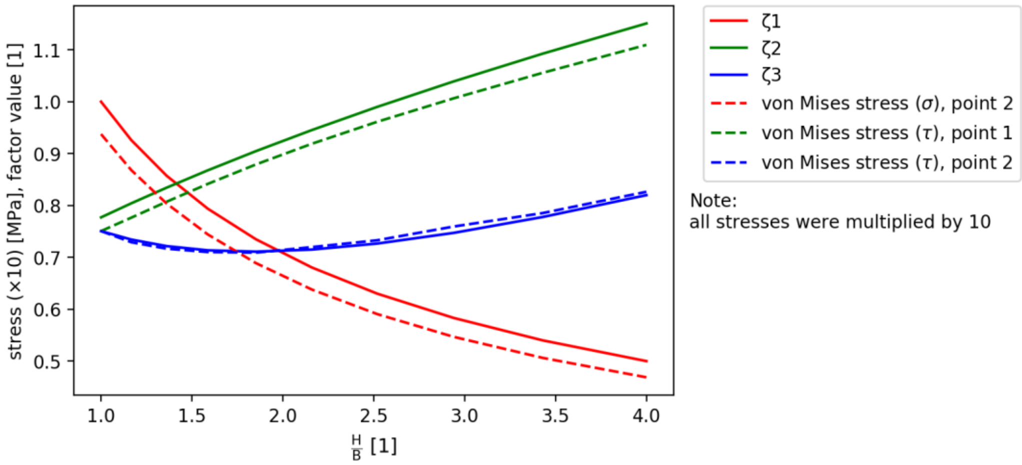


Fig. 38 Calculated stress compared to factors prediction

Relative size of bending moment and torsion moment described by coefficients:

$$k_b = \frac{M_b}{M_b + M_t} \tag{40}$$

$$k_t = \frac{M_t}{M_b + M_t} \tag{41}$$

Formula of combined bending-torsion shape factor based on von Mises yield criterion (equation which multiplies the set makes relative ratio 10 against von Mises stress):

$$\zeta_c = \max\left\{\sqrt{3 \cdot \left(\zeta_2 \frac{k_t}{\sqrt{3}}\right)^2}, \sqrt{(\zeta_1 \cdot k_b)^2 + 3 \cdot \left(\zeta_3 \cdot \frac{k_t}{\sqrt{3}}\right)^2}\right\} \cdot \sqrt{1+3} \tag{42}$$

Equation bellow is shape factor for beam loaded by combination of bending moment and torsion moment (in physics mean, lower value is better for design):

$$\zeta_c = 2 \cdot \max\left\{k_t \cdot \left(\frac{H}{B} + 1\right)^{\frac{3}{7}}, \sqrt{\left(k_b \cdot \left(\frac{H}{B}\right)^{-0.5}\right)^2 + \left(k_t \cdot \left(\frac{\frac{H}{B} + \left(\frac{H}{B} - 2\right)^2}{5 \cdot \left(\frac{H}{B} + 1\right)} + 1.1\right)\right)^2}\right\} \tag{43}$$

where M_b is bending moment and M_t torsion moment. Combined bending-torsion factor in the form of prof. Ashby form can be calculated using equation:

$$\phi_{BT}^{f} = \frac{1}{\zeta_c} \tag{44}$$

Equation of the combined shape factor is relatively complicated, but it can be used to create material maps (modified by shape factors [1]) or to find an optimal shape in first stages of design process.

For example, to find an optimal ratio for a one end fixed beam with rectangle cross section, if load:

- bending moment M_{b1} = 1000 Nm
- torsional moment M_{t1} = 2000 Nm

There are two possible solutions – to find minimum on ζ_c indicator using numerical equation solver or draw graph of ϕ_{BT}^{f}:

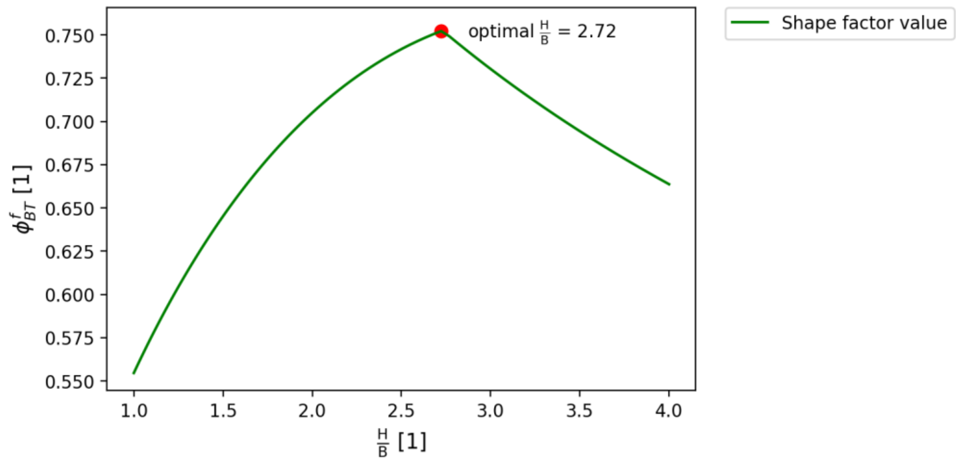


Fig. 8 Solution of the example of combined load, optimal solution marked by red point

The picture above explains that optimal cross section parameters ratio is around 2.72, the next step is to estimate cross section area size, calculate H and B dimensions and continue design process.

Conclusion

One of the goals of this paper was to show what the shape factors and materials indices are, how they works and why they are beneficial. As the next goal was to try to find out shape factor of combined stress compounded from bending and torque.

As is obvious from last part of this paper, is possible to create a shape factor for combined stress. But whole method is complicated and it takes some time. For this reason is suitable to creating complex shape factors only for special used where is possible to used it many times. For other applications is still better to use FEM analysis.

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