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Short overview of current methods for detecting fatigue limit

KAĽAVSKÝ Adam[1, a], HUŇADY Róbert[1, b] and KICKO Michal[1, c]

[1] Department of Applied Mechanics and Mechanical Engineering, Technical University of Košice, Faculty of Mechanical Engineering, Slovakia

[a]adam.kalavsky@tuke.sk, [b]robert.hunady@tuke.sk, [c]michal.kicko@tuke.sk

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Abstract. The paper deals with determining the fatigue limit of a casual construction material. The paper described two different approaches to do it. The first approach is based on analytical calculation of fatigue limit evaluated by theory of elasticity and plasticity. However, these principles provide only a very rough estimate of the fatigue limit. In practice, the experimental principles described in the second part of the article are preferred and used more often.

Introduction

Machine components, structures, automobiles, planes, etc. are often subjected to a cyclic loading. Cyclic loading very often leads to the microscopic material damage. If the stress caused by this load is less than the strength of the material, the damage will accumulate until a crack develops or other damage leads to fracture of the component. Such a process of accumulation of failure that leads to fracture of a component due to cyclic loading is called fatigue [1] . Material fatigue has caused many catastrophic failures of planes, ships, bridges and other structures in history [2] , [3] . In the past fatigue curves (S-N curves) were discovered, for which the specific load amplitude (stress, plastic deformation, etc.) was called fatigue limit and it was assumed that there was no fatigue fracture below this value. The example of fatigue curve can be seen in Fig. 1.

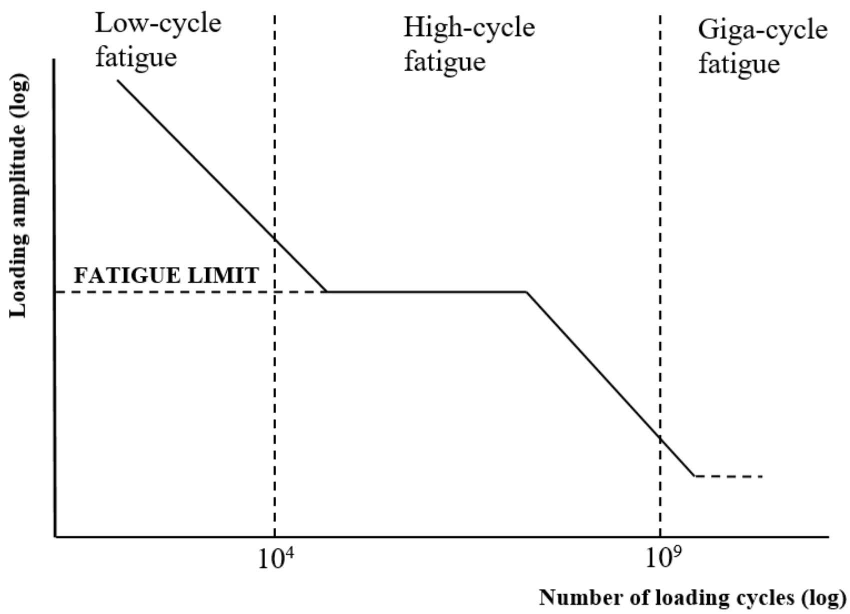

Fig. 1 – S-N curve

1. Analytic methods

This part shows how a design engineer could estimate the fatigue lifetime duration for a mechanical system (without knowing of a fatigue diagram). In Table 1 are listed basic formulas to estimation fatigue limit based on relation to main material characteristics like Yield Limit, Strength Limit, Hardness, True Fracture Strength.

Table 1 – Analytical evaluation of fatigue limit [4]

Evaluating value	Formula (R = -1, P = 50%) [MPa]	Condition of validity	Author
Strength Limit (R_m)	$\sigma = 0.46 \cdot R_m$ $\tau = 0.27 \cdot R_m$	Steel $R_m \geq 1200$ MPa	Žukov
Yield Strength (R_e, τ_k)	$\sigma = 0.45 \cdot R_e + 122$ $\tau = 0.448 \cdot \tau_k + 52$	Construction steel Construction steel	Žukov Buch
True Fracture Strength (σ_f)	$\sigma = 0.35 \cdot \sigma_f - 10$	Construction steel	Žukov
Hardness (HB)	$\sigma = (0.128 \ldots 0.156) \cdot HB$	Carbon steel	Grebenik
$R_m + R_e$	$\sigma = 0.285 \cdot (R_e + R_m)$	Construction steel	Šapošnikov

Example:
Material properties of SAE-AISI 1040 Steel are known as $R_e = 460$ MPa and $R_m = 640$ MPa. Problem formulation is to find the fatigue limit of investigated material by analytical calculation.

The SAE-AISI 1040 belongs to construction steel family. Therefore, formulas by Žukov and Šapošnikov can be used at problem solution (see Table 1).
$\sigma = 0.45 \cdot 460 + 122 = 329$ MPa. … Žukov
$\sigma = 0.285 \cdot (Re + Rm) = 0.285 \cdot (460 + 640) = 313.5$ MPa … Šapošnikov
Estimated fatigue limit should be close to the values 313.5 and 329MPa.

Experimental methods

In the engineering practice, the most used experimental methods based on statistics are staircase method and their modified version. Among other statistical methods for experimental evaluation of fatigue (strength) limit belong the methods listed in Table 2.

Table. 2 – Overview of fatigue strength evaluation methods.

Group	Method
Conventional	• S-N test method
Quantal response tests	• Probit method • Staircase method • Two-point method • Arcsin√P method
Accelerated stress tests	• Prot method • Step-loading method
More advanced statistical methods	• Random fatigue limit model • Bayesian method • Bootstrap method

Quantal response tests

Unlike the conventional S-N approach, quantal response tests do not consider the actual time to failure, but rather they are given by their nature a “pass/fail” approach. Thus, quantal response tests are designed to handle runout test data. In a quantal response test, specimens are tested at a certain load level and they either survive the specified number of cycles or they fail. Statistical analysis of test results allows an estimate of median and standard deviation of the fatigue strength at the specified number of cycles based on the proportion of failed specimens at each load level.

Staircase method

Fatigue limit σ_e is estimated by staircase method from Eq. 1 [5]

$$\sigma_e = \sigma_0 + d \cdot \left(\frac{A_n}{\sum n_i} - \frac{1}{2} \right), \tag{1}$$

where n_i is number of events (failures or survivals) for each stress level. Range between two stress levels is expressed by d . Lowest stress level is denoted as σ_0 . Quantity A_n is obtained from

$$A_n = \sum i n_i \ . \tag{2}$$

In the staircase method, specimens with progressively increasing stress amplitude are tested until the failure occurs.

Example:

Problem formulation is to find the fatigue limit of investigated material at 10^7 cycles by staircase method. The results of fatigue tests of SAE-AISI 1040 Steel are shown in Table 3.

Table 3 – Dataset of SAE-AISI 1040 Steel used in staircase method.

Stress level σ_i [MPa]	#																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
325							x												x	
320				x		o		x										o		o
315	x		o		o				x				x		x		o			
310		o								x		o		o		o				
305											o									
x	fail																			
o	pass																			

At the first step, it is necessary to perform data analysis and fill the values in Table 4.

Table 4 – Data extracted from Table 3.

Stress level	Level	Values	
σ_i [MPa]	i	n_i	$i \cdot n_i$
325	3	2	6
320	2	2	4
315	1	4	4
310	0	1	0
Σ	–	9	14

Subsequently, the average value of fatigue limit is calculated from Eq. 1

$$\sigma_e = 315 + 5 \cdot \left(\frac{14}{9} - \frac{1}{2} \right) = 314.72 \text{ MPa.}$$

Using the Matlab software, it is possible to obtain a curve of normal distribution of fatigue limit (Fig. 2). The normal distribution is calculated by standard formula as follows

$$f(\sigma_f) = \frac{1}{7.6349 \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\sigma_i - 314.7222^2)}{2 \cdot 7.6349}},$$

where σ_i acquire values between lower fatigue limit and average fatigue limit in sum with estimated standard deviation.

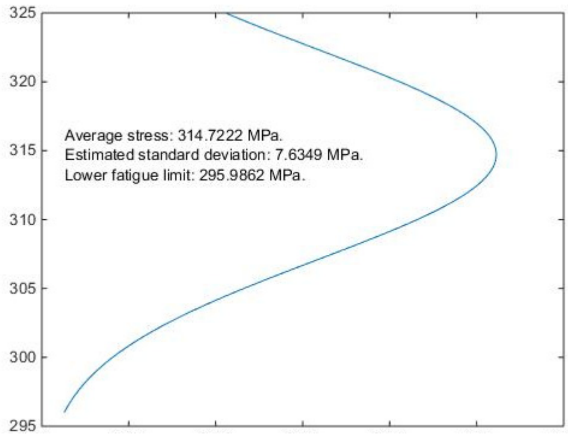

Fig. 2 – Normal distribution of fatigue limit values.

Average level of fatigue limit acquired from measured dataset achieved the value of 314.72 MPa. Estimated standard deviation was calculated at 7.63MPa and lower fatigue limit had approximated value ca. 296MPa (for 10% probability of failure at 95% confidence level).

Modified staircase method

Modified staircase method can be used only in the case if the standard deviation of fatigue strength is known (e.g. it is estimated from S-N curve). Minimum six specimens are necessary for testing in order to reliability of modified staircase method to be preserved. Optimum test is performed at 15 specimens [6] . Fatigue limit σ_e is estimated by staircase method from Eq. 3

$$\sigma_e = \frac{\sum_{i=2}^{n+1} \sigma_i}{n}, . \qquad \textbf{(3)}$$

where σ_i is stress level, and the number of experiments is denoted as n. In the process of measurement the value of σ_{i+1} is not obtained, but it is specify from previous stress level.

Example:
Problem formulation is to find the fatigue limit of investigated material at 10^7 cycles by modified staircase method. The results of fatigue testing of SAE-AISI 1040 Steel is shown in the Table 3.

Table 5 – Dataset of SAE-AISI 1040 Steel used in modified staircase method.

Level i	Measurement rank								
	1	2	3	4	5	6	7	8	9
σ_i [MPa]	315	310	315	320	315	320	325	320	315
Result	x	o	o	x	o	o	x	x	*
x fail									
o pass									
* the result of test was specify from previous stress level									

$$\sigma_e = \frac{(310 + 315 + 320 + 315 + 320 + 325 + 320 + 315)}{8} = 317.5\,\mathrm{MPa}.$$

Conclusions

The paper presented a brief overview of current methods to determine fatigue limit using different analytical and experimental approaches. For analytical solution Žukov and Šapošnikov model of evaluation fatigue limit was used. Statistical model used in experimental method was Staircase method and its modified version. Deference of results between experimental and analytical model was under 4%. It may be noted that the analytical solution was quite accurate.

Acknowledges

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References

[1] F. Trebuňa, M. Buršák. Medzné stavy - lomy. Košice : TUKE, 2002. p. 348.

[2] F. Trebuňa, P. Bigoš, et.al. Determination of reasons of cracking of outside shell of converter hood by means of experimental analysis methods of strength. Olomouc: Palacký University, EAN, 1997, p. 217-222.

[3] P. Bigoš, F. Trebuňa, E. Faltinová. Ussage of stress damage cumulation theories for assesment of heavz metallurgical bridge crane steel structure conditions. Košice: TUKE, 1997, P. 8-14, ISSN 1335-2334.

[4] M. Růžička, M. Hanke, M Rost. Dynamická pevnosť a životnosť. Praha : ČVUT, 1987. p. 213.

[5] S.R. Schmid, B.J. Hamrock, B.O. Jacobson. Fundamentals of Machine Elements. Third. Boca Raton : Taylor & Francis Group, 2014. p. 610.

[6] ČSN ISO 12107:2013. Kovové materiály – Zkoušky únavy – Statistické plánování a analýza údajů.