

Prediction of the tensile breaking force and extension at break of woven fabrics in an arbitrary load direction

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Abstract. Anisotropy is the characteristic which is typical for most materials, specially woven fabrics. Influence of direction of tensile force acting on the properties of the fabric is big and frequently tested. The woven fabric can be defined as orthogonal elastomer. This paper presents an experimental testing and determination of tensile breaking forces, extension at break and ultimate strength of woven fabric for different angles of force direction. Cotton fabric samples in plain weave were tested under tensile forces at the angles of 0°, 15°, 30°, 45°, 60°, 75° and 90° with respect to the weft direction. Based on the experimentally obtained values, theoretically calculated breaking forces, extension at break and ultimate strength for arbitrarily chosen fabric directions was calculated. This paper compares experimental and theoretical values of breaking forces, extension at break and ultimate strength. Experimental results show the applicability of theoretical equations for calculating the breaking force and extension at break for arbitrarily chosen direction of cutting the fabric specimen. Therefore, the measurements need to be implemented when the tensile force acting on the fabric only in the warp (90°), weft (0°) and at angle of 45°.

Introduction

Woven fabrics are generally non homogeneous, anisotropic and discontinuous objects. Special types of anisotropic materials having two mutually perpendicular planes of elastic symmetry are called orthotropic materials. These planes of elastic symmetry are planes of orthotropy, and their cross sections are axes of orthotropy (x and y) [1]. So the woven fabric is elastic orthotropic material. In a biaxial woven structure two main directions are defined: longitudinal (warp, y-axes) and transverse (weft, x-axes). Theoretical analysis of fabric behaviour is often very complex, so that the experimental verification of theoretical predictions for them is more important than for other materials. The method for measuring anisotropic tensile properties of fabrics is called "uniaxial test method" or the method of force action in only one direction [2]. When the angle of an external load (tensile force) changes, tensile breaking forces and extension at break change too. The aim of this work is to analyse the impact of anisotropy and different weaves on the tensile breaking forces and extension at break of woven fabrics in various directions and to determine the degree of agreement between experimental results and calculated obtained values.

Anisotropy of breaking force and extension at break

When the tensile force F is applied to the woven fabric in arbitrary directions, the normal stresses are varied depending on the direction of the force acting. The highest values of these tensile forces which cause breakage of the fabric are breaking tensile force F_φ (N). Eq. (1a) and (1b) calculate the values of extension at break for any combination of reversed axes in the plane of orthotropy [3, 4].

$$\varepsilon_\varphi = \frac{\varepsilon_0}{\cos^4 \varphi + b \cdot \sin^2(2 \cdot \varphi) + c \cdot \sin^4 \varphi} \quad (1a)$$

$$\text{where are: } \varepsilon_\varphi = \varepsilon_k, \varepsilon_y = \varepsilon_{90}, \varepsilon_x = \varepsilon_0, b = \frac{\varepsilon_0}{\varepsilon_{45}} - \frac{1 + \varepsilon_0/\varepsilon_{90}}{4}, c = \frac{\varepsilon_0}{\varepsilon_{90}}. \quad (1b)$$

The angle φ is the angle between the axis x (weft direction) and direction of tensile force acting [4, 5]. ε_φ is the value of extension at break for the arbitrary direction of force acting relative to the weft of the fabric, ε_0 , ε_{90} are the extension at break in the orthotropy axes (weft and warp direction), and ε_{45} is the extension at break at angle $\varphi=45^\circ$.

Theoretical breaking tensile forces F_φ (N) of orthotropic materials on the directions of all angles φ can be determined by measured values of breaking tensile forces in the directions at an angle of 0° , 45° , 90° by using the expressions (2a) and (2b) [3, 4]:

$$F_\varphi = \frac{F_0}{\cos^4 \varphi + b \cdot \sin^2(2 \cdot \varphi) + c \cdot \sin^4 \varphi} \quad (2a)$$

$$\text{where are: } F_\varphi = F_k, F_y = F_{90}, F_x = F_0, b = \frac{F_0}{F_{45}} - \frac{1 + F_0/F_{90}}{4}, c = \frac{F_0}{F_{90}}. \quad (2b)$$

The values of breaking forces F_0 , F_{45} , F_{90} are obtained in laboratory tests under uniaxial stretching of the sample when the tensile forces acting in the directions $\varphi = 0^\circ$, 45° , 90° .

The second way of calculating the values of breaking forces is that on the basis of the experimental values of measured breaking forces of fabric samples tested at different angles, sets the relationship between force F and angle φ . The relationship is shown in Eq. (3) [5]:

$$F(\varphi) = a\varphi_i^2 + b\varphi_i + c \quad (3)$$

The mathematically most acceptable approach is to customize the experimental data to the quadratic equation-the polynomial second order equation (3) [5]. The condition must be met:

$$\sum_{i=0}^n (a\varphi_i^2 + b\varphi_i + c - F_i)^2 = \min \quad (4)$$

where a , b i c - the second degree polynomial parameters. Partial derivation of the Eq. (4) by a , b , and c and by solving the resulting system, the second degree polynomial parameters are obtained. From the Eq. (3) the theoretical values of the breaking forces are obtained.

Based on the experimental values of extension at break, the relationship between extension and the angle of force action is set by Eq. (5):

$$\varepsilon(\varphi) = a\varphi_i^2 + b\varphi_i + c \quad (5)$$

Again can be used quadratic function to fit the results.

$$\sum_{i=0}^n (a\varphi_i^2 + b\varphi_i + c - \varepsilon_i)^2 = \min \quad (6)$$

Partial derivation of the Eq. (6) by a , b , and c and by solving the resulting system, the second degree polynomial parameters are obtained. From the Eq. (5) the theoretical values of the extension at break are obtained.

Experimental determination of breaking force and extension at break

The experiment was carried out by measuring fabric spatial deformation under the action of tensile force till rupture. Two different cotton fabrics of different weaves (plain and twill) were used. Their structural properties are shown in Table 1. Width of each specimen is $b=50$ mm.

Table 1: Description of fabrics

Fabric structure	Warp direction			Weft direction			Weight (g/m ²)	Thickness fabric (mm)
	Yarn fibres	Yarn count (tex)	Density (cm ⁻¹)	Yarn fibres	Yarn count (tex)	Density (cm ⁻¹)		
Plain	Cotton	36	29,3	Cotton	36	20,0	193,7	0,48
Twill 1/3	Cotton	36	29,3	Cotton	36	20,0	191,6	0,53

Table 2: Mean experimental, theoretical and fitted values of breaking force and extension at break for plain weave fabric

φ (°)	ε (%)	$\varepsilon_{\text{calc}}$ (%)	$\varepsilon_{\text{poly}}$ (%)	F (N)	F_{calc} (N)	F_{poly} (N)
0	18,24	18,24	12,87	145,3	145,3	97,0
15	22,12	20,13	23,49	198,4	161,0	213,3
30	26,01	26,53	32,17	251,5	216,4	308,7
45	38,29	38,29	38,91	330,9	330,9	383,0
60	42,30	49,72	43,72	444,6	478,9	436,3
75	46,57	51,97	46,59	497,5	548,1	468,6
90	50,84	50,84	47,53	550,4	550,4	479,9
105	48,16	51,97	46,54	501,1	548,1	470,1
120	45,47	49,72	43,60	451,8	478,9	439,4
135	38,29	38,29	38,74	339,2	330,9	387,6
150	32,15	26,53	31,94	261,7	216,4	314,8
165	26,01	20,13	23,20	203,0	161,0	221,1
180	10,41	18,24	12,53	151,3	145,3	106,2

Yarn linear density was determined by the gravimetric method according to standard ISO 2060:1994. Number of threads per unit length was determined according to standard ISO 7211-2:1984. Standard ISO 5084:1996 describes a method for the determination of the thickness of fabric. Before testing all specimens were conditioned under the conditions of standard atmosphere (relative air humidity $65 \pm 2\%$, at a temperature of $20 \pm 2^\circ\text{C}$). For the purposes of this testing standard specimens with dimensions 300×50 mm were clamped in clamps of the tensile tester at a distance of 200 mm and pulling speed: 100 mm/min and subjected to uniaxial tensile load till rupture. The specimens were cut in seven different directions: warp direction ($\varphi=90^\circ$), weft direction ($\varphi=0^\circ$), and at angles 15° , 30° , 45° , 60° , 75° to the weft. Three tests were done for each mentioned direction of force action on the fabric specimen. Tensile

properties of all specimens were tested according with standard ISO 13934-1:2008 using the strip method for measuring fabric strength on a tensile strength tester Textechno Statimat M [6]. The measurement results were collected and stored on the hard disk by a computer program of tensile strength tester. The experimental mean values of breaking force F_{ϕ} (N) and the extension at break ε_{ϕ} (%) are shown in Tab. 2 and Tab. 3

According to Eq. (2) and based on the experimental data $F_{0^{\circ}}$, $F_{90^{\circ}}$, $F_{45^{\circ}}$, in Table 2 and Table 3, theoretical values of breaking force F_{calc} were calculated for various angles of tensile force action. According to Eq. (3), the fitted values of the breaking force Poly. (F (N)) was obtained. According to Eq. (1) and based on the experimental data $\varepsilon_{0^{\circ}}$, $\varepsilon_{90^{\circ}}$, $\varepsilon_{45^{\circ}}$, in Table 2 and Table 3, theoretical values of extension at break $\varepsilon_{\text{calc}}$ were calculated According to Eq. (5), the fitted values of extension at break Poly. (ε (%)) was obtained. The experimental and calculated values for plain weave fabric are shown in Tab. 2, and for twill weave fabric are in Tab. 3.

Table 3: Mean experimental, theoretical and fitted values of breaking force and extension at break for twill weave fabric

ϕ ($^{\circ}$)	ε (%)	$\varepsilon_{\text{calc}}$ (%)	$\varepsilon_{\text{poly}}$ (%)	F (N)	F_{calc} (N)	F_{poly} (N)
0	16,10	16,10	13,08	131,2	131,2	90,7
15	21,45	17,81	22,52	179,8	145,3	195,5
30	26,90	23,69	30,25	240,7	195,0	281,6
45	34,68	34,68	36,27	298,1	298,1	348,8
60	41,60	45,47	40,58	405,4	434,5	397,4
75	42,59	47,34	43,17	457,4	503,1	427,1
90	46,12	46,12	44,06	507,8	507,8	438,1
105	43,56	47,34	43,24	436,5	503,1	430,3
120	38,40	45,47	40,71	412,2	434,5	403,7
135	35,48	34,68	36,47	308,1	298,1	358,4
150	29,11	23,69	30,51	267,5	195,0	294,4
165	24,70	17,81	22,85	200,9	145,3	211,5
180	9,52	16,10	13,48	138,7	131,2	109,9

The diagram of experimental and calculated values of breaking forces for plain weave fabric for each 15° is shown in Fig 1a. The diagram of experimental and calculated values of extension at break for plain weave fabric for each 15° is shown in Fig 1b.

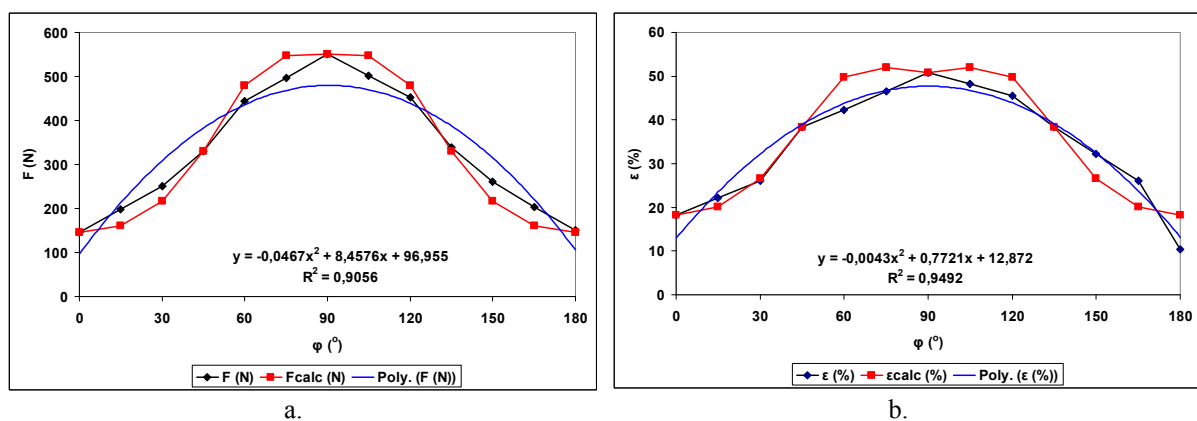


Fig. 1: Diagram of experimental and calculated values for plain weave fabric: a) breaking forces, b) extension at break

The diagram of experimental and calculated values of breaking forces for twill weave fabric for each 15° is shown in Fig 2a. The diagram of experimental and calculated values of extension at break for twill weave fabric for each 15° is shown in Fig 2b.

In Fig. 1 and Fig. 2 is evident that woven fabrics of both weaves have the lowest value of breaking forces and extension at break in the transverse or weft direction ($\varphi = 0^\circ$, $\varphi = 180^\circ$). The values of breaking forces and the values of extension at break gradually increase from the weft direction ($\varphi=0^\circ$) and the highest values are achieved in the longitudinal or warp direction ($\varphi = 90^\circ$). Diagrams in Fig. 1 and Fig. 2 have symmetric values in relation to bias angle $\varphi = 90^\circ$. The values of breaking force for plane weave fabric samples in the longitudinal direction ($\varphi = 90^\circ$) is higher by about 3.8 times in relation to the transverse direction ($\varphi = 0^\circ$), and in relation to the diagonal direction ($\varphi = 45^\circ$) is higher by 1.7 times. The values of extension at break for plane weave fabric samples in the longitudinal direction ($\varphi = 90^\circ$) is higher by about 2.8 times in relation to the transverse direction ($\varphi = 0^\circ$), and in relation to the diagonal direction ($\varphi = 45^\circ$) is higher by 1.3 times.

The values of breaking force for twill weave fabric samples in the longitudinal direction ($\varphi = 90^\circ$) is higher by about 3.9 times in relation to the transverse direction ($\varphi = 0^\circ$), and in relation to the diagonal direction ($\varphi = 45^\circ$) is higher by 1.7 times. The values of extension at break for twill weave fabric samples in the longitudinal direction ($\varphi = 90^\circ$) is higher by about 2.9 times in relation to the transverse direction ($\varphi = 0^\circ$), and in relation to the diagonal direction ($\varphi = 45^\circ$) is higher by 1.3 times.

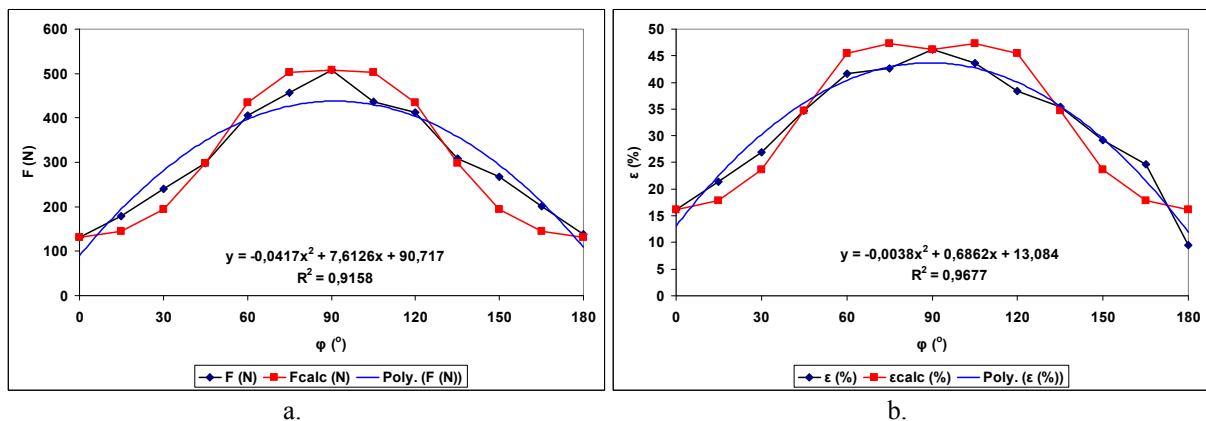


Fig. 2: Diagram of experimental and calculated values for twill weave fabric: a) breaking forces, b) extension at break

Conclusions

The values of breaking force and extension at break are changed depending on the direction of force acting. Prediction of breaking forces and extension at break can be carried out in two ways. Theoretical equation with high accuracy can be used to calculate values of breaking forces and extension at break of fabrics for an arbitrarily chosen direction of action of tensile force. The values of breaking force and extension, also can be obtained by fitting the experimental values with the quadratic equation. A good agreement between experimental results and the calculated obtained values of breaking forces and extension at break was shown. Therefore, the measurements need to be implemented when the tensile force acting on the fabric only in the warp, weft and at an angle of 45° .

In plain weave fabric, a very high coefficient of determination R^2 ($R^2 = 0.9056$) is obtained, which means that there is almost an exact dependence between the value of breaking force F obtained by quadratic function and angle φ . For the dependence between extension at break ϵ obtained by quadratic function and angle φ , the coefficient of determination is $R^2 = 0.9492$.

In twill weave fabric, a very high coefficient of determination R^2 ($R^2 = 0.9158$) is obtained, which means that there is almost an exact dependence between the value of breaking force F obtained by quadratic function and angle φ . For the dependence between extension at break ε obtained by quadratic function and angle φ , the coefficient of determination is $R^2 = 0.9677$.

The values of breaking force and extension of plain weave fabric are about 10% higher than the values of breaking force and extension of twill.

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