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GRAPHICAL METHOD OF CONSTRUCTION OF MOHR´S CIRCLE USING MOIRE FRINGES GRADIENTS ANT ITS APPLICATION

GRAFICKÁ METOSDA KONSTRUKCE MOHROVY KRUŽNICE S VYUŽITÍM GRADIENTŮ MOIRE INTERFERENČNÍCH PRUHŮ A JEJÍ APLIKACE

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The paper deals with the possibility of Mohr´s circle construction using moiré-fridges gradients. The presented method was applied to the bar of rectangular cross-sectional area with oblique through thickness crack under loading by force acting along the bar axis.

Keywords

Moire method, fringe gradient, strains, Mohr´s circle

Using moire method one obtains a field of moire fringes – isothetics. The magnitudes of the corresponding displacement component along two neighbour fringes differ each other by value equal to the master – grating pitch p. The position of a point of a plain body under loading is changing. The new position of the point can be expressed by the displacement vector u with components u_i, $i = x, y$ in the case of plane stress - state. Then the components of the strain tensor are

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{1}$$

The graphical construction of the Mohr´s circle at a point

The obvious is a method of computing of derivatives of the displacements in sections parallel to the coordinate axes. Another possibility is to use the fringe gradient. This approach can be used in the case of the construction of Mohr´s circle at a point, when strains and rotations are small.

Let us consider a field of moiré fringes of the $u_x = u$ family in the crack tip vicinity with a point A between two fringes of order k and k +1, Fig. 1.

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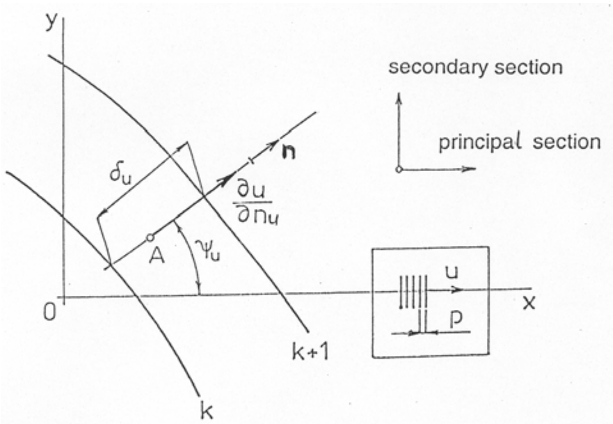

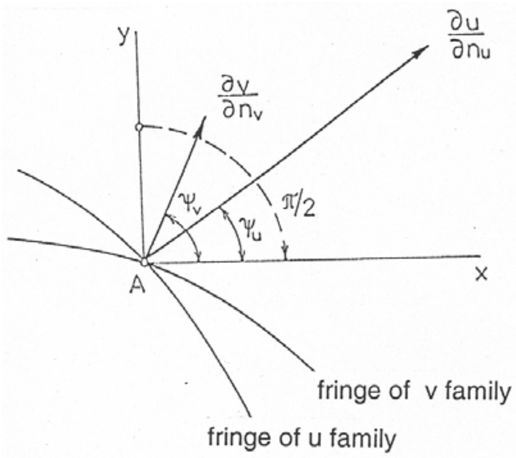

Fig.1 Fig. 2

To find the fringe gradient at a point *A*, a section of the displacement surface $u_i = u_i(xy)$ in the direction of the gradient has to be used. Let us take into account the gradient of the common normal **n** to neighbour fringes ordered as k_n and k_{n+1} for moiré fringes of $u_x = u$ family. Let us designate the distance between those fringes measured along the common normal as δ_u, in general δn_i, for u_x and u_y families. Then the gradients of displacement components u_i in the plane *x, y* will be

$$grad\ u_i = \frac{\partial u_i}{\partial n_i} = \frac{p}{\delta_{n_i}} \tag{2}$$

Let us represent these gradients by vectors with angles $\psi_{u\,i}$ between vectors and the *x*-axes.

Let us consider strain components, which in general represent the derivatives of the displacement components in the principal sections and the sum of the derivatives in the directions of secondary sections

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{3}$$

To simplify the designation of quantities $u = u_x$, $v = u_y$ was used. Then the tangent of the angle $\psi_{u\,i}$ between the x –axis and the vector of $grad\, u_i$ is given by

$$tg\,\psi_{u_i} = \frac{\partial u_i / \partial y}{\partial u_i / \partial x} \tag{4}$$

Let us consider a vector representation of both gradients in a physical plane, Fig. 2.

For transition from the physical plane with the origin O' to the $\varepsilon - \gamma / 2$ (direct and shear strain components) plane, the y - axis has to be rotated clockwise $\pi / 2$. Then $\dfrac{\partial v}{\partial n_v}$ vector rotates being rigidly fixed to the y – axis, fig. 3. For the Mohr′s circle construction the ε - axis is parallel to the x – axis passing the point *V*. Then the points *U* and *V* of the vectors $\dfrac{\partial u}{\partial n_u}$ and $\dfrac{\partial u}{\partial n_v}$ are joined and the vector *VU* is obtained. Projections of these vectors *VB* and *UB* are equal to

$$VB = \frac{\partial u}{\partial n_u}\cos\psi_u - \frac{\partial v}{\partial n_v}\sin\psi_v \tag{4}$$

By the Fig. 4

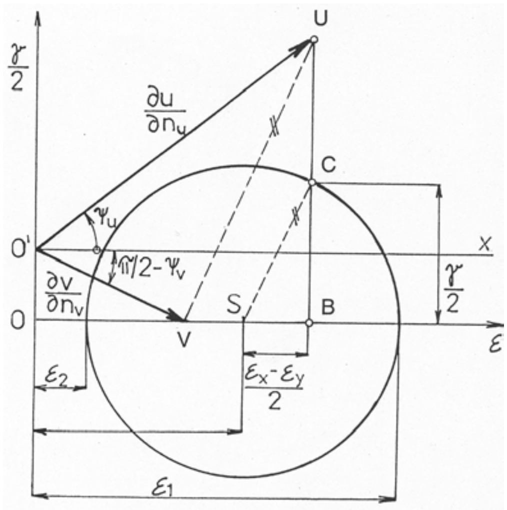

Fig. 3

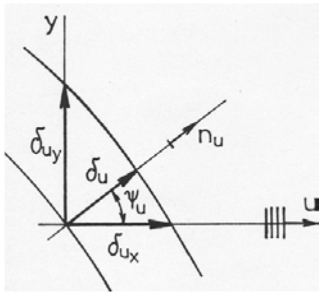

Fig. 4

$$\varepsilon_x = \frac{p}{\delta_{u_x}}\text{ , where } \delta_{u_x} = \frac{\delta_u}{\cos\psi_u} \tag{5}$$

After inserting and treating one obtains

$$\varepsilon_x = \frac{p}{\delta_u}\cos\psi_u = \frac{\partial u}{\partial n_u}\cos\psi_u \tag{6}$$

Similarly

$$\varepsilon_y = \frac{p}{\delta_v}\sin\psi_v = \frac{\partial v}{\partial n_v}\sin\psi_v \tag{7}$$

Taking into account equations (6) and (7) will be

$$VB = \varepsilon_x - \varepsilon_y \tag{8}$$

From the properties of Mohr´s circle, point *S* which is located half distance between *V* an *B,* is the centre of Mohr´s circle.

Let us express the projection *UB* of the vectors $\dfrac{\delta u}{\delta n_u}$ and $\dfrac{\delta v}{\delta n_v}$ respectively. We obtain

$$UB = \frac{\partial u}{\partial n_u}\sin\psi_u + \frac{\partial v}{\partial n_v}\cos\psi_v \tag{9}$$

From Fig. 4 will be

$$\frac{\partial u}{\partial y} = \frac{p}{\delta_{u_y}} = \frac{p}{\delta u}\sin\psi_u \qquad \frac{\partial v}{\partial x} = \frac{p}{\delta_{v_x}} = \frac{p}{\delta v}\cos\psi_v \tag{10}$$

After comparison of equations (10) and (9) it is obvious, that UB

$$UB = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy} \tag{11}$$

Then the radius of the Mohr´s circle is obtained by joining point S with point C which is half way between U and B. The origin O of the coordinate system $\varepsilon - \pi/2$ is the point of intersection of the straight line VS and the vertical axis. It is obvious that distance *SC,* where the point C is located half way between B and U, is the radius of Mohr´s circle

$$SC = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \tag{12}$$

Analysing the previous procedure it is obvious, that the diameter of the circle is parallel to the abscissa *VU.*

Flat bar with an oblique crack

The method was applied to the bar of rectangular cross sectional area of the width 30 mm and thickness of 3 mm with oblique through thickness crack with slope of $\alpha = 34^\circ$ to the x – axis. The crack was modelled by the narrow notch of the length 6 mm modelled as a thin saw cut. The specimen was made from low-carbon steel of yield stress $\sigma_y = 437\,MPa$.

Moire method was applied for experimental strain analysis. The advantage of that method is that it provides the field of moiré fringes in the whole zone surrounding the crack tip. The two families of moiré fringes for loading of the specimen by the force $F = 29\,kN$ are presented in Fig. 5. There is u_x fringes family on the left, when the grid of pitch $p = 0{,}0254\,mm$ was used and family of u_y displacement component is on the right for grating of pitch $p = 0{,}0508\,mm$.

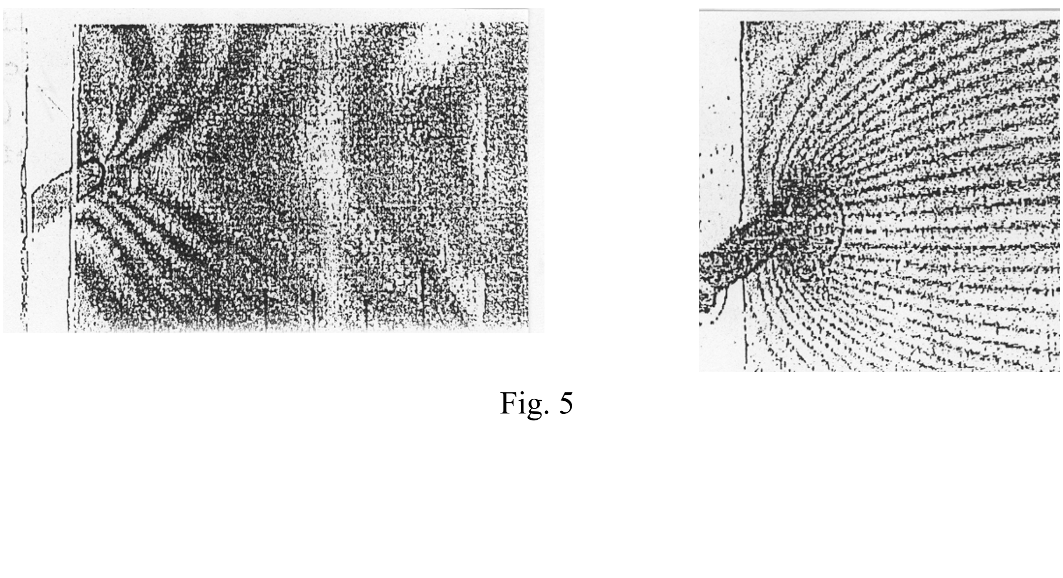

Fig. 5

There are presented strain components ε_x, ε_y, $\gamma'_{xy} = \dfrac{\partial u}{\partial y}$, $\gamma''_{xy} = \dfrac{\partial v}{\partial y}$ and γ_{xy} along the cross-section in the distance $y = 5\ mm$, in Fig. 6.

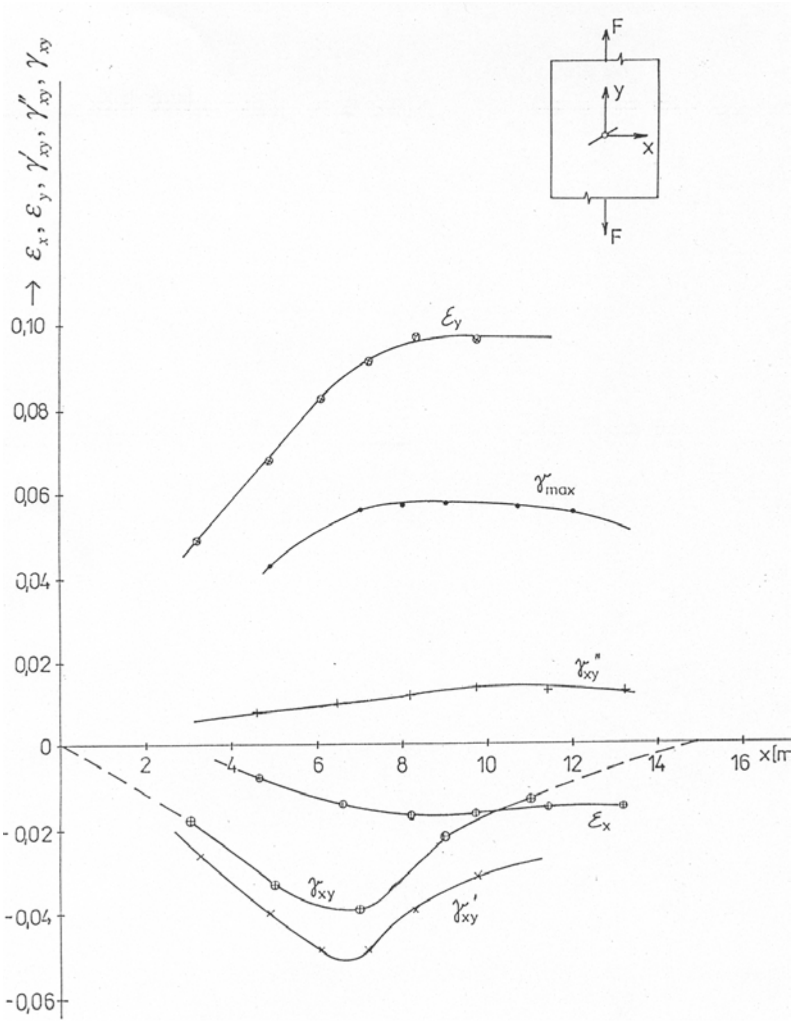

Fig. 6

Construction of the Mohr´s circle for the point in position $x = 7\ mm$ and $y = 5\ mm$ is presented in Fig. 6. Using corresponding fields of moiré fringes the following magnitudes of gradients were calculated $\dfrac{\partial u}{\partial n_u} = 0{,}11$ and $\dfrac{\partial v}{\partial n_v} = -0{,}023$.

The strain components $\varepsilon_x = -0{,}015$, $\varepsilon_y = 0{,}009$ and $\gamma_{xy} = -0{,}039$ were determined at chosen point using obvious procedure. From the Mohr´s diagram one obtains principal strain $\varepsilon_1 = 0{,}093$ and $\varepsilon_2 = -0{,}019$. Comparing these values with those obtained in the graph in Fig. 7 there is very good agreement.

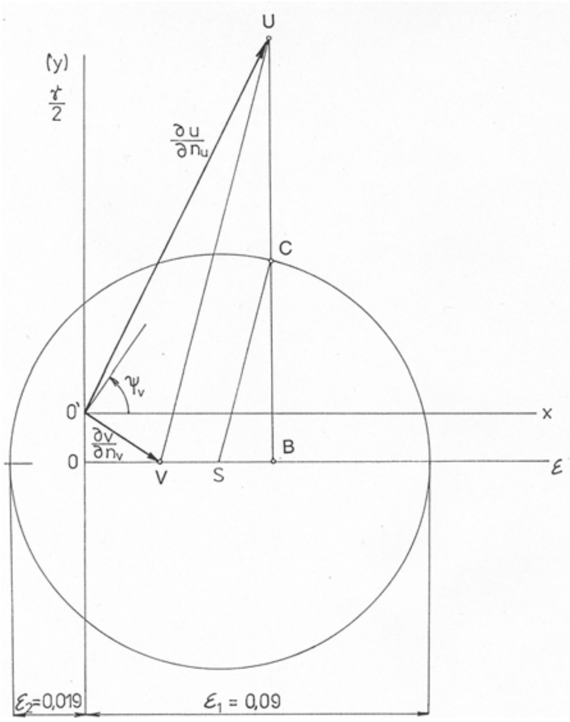

Fig. 7

Conclusion

The method was applied to the flat bar with the oblique central crack. The presented construction of Mohrs circle was appeared very convenient, but it accuracy is significantly influenced by the accuracy of determinations of angles of isothetics in the taking into account point. It means it depends on the accuracy of determination of the mean curves of moire fringes.

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